



# Total Portfolio Performance Attribution Methodology

Morningstar Methodology Paper  
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## Version History

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May 2013	Updated for the addition of Premium/Discount, Contribution, and Average Weightings sections.
September 2009	Original version.

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# Introduction

## **Total Portfolio**

Total portfolio refers to a portfolio with a strategic asset-allocation policy consisting of multiple asset classes implemented with various investment managers. This is a typical portfolio for pension, endowment, and foundation funds, some individual investors, and so on. It is also applicable to other portfolios with a defined strategic asset-allocation policy such as target-date funds. The term for this type of portfolio is total portfolio or total fund<sup>1</sup> because it is a total investment portfolio that includes all of an investor's assets, as opposed to an equity or fixed-income portfolio of individual securities that constitutes a slice of an overall asset allocation. Another commonly used term is balanced portfolio.<sup>2</sup> The investment decision-maker is usually referred to as the "plan sponsor," a term that originated in pension fund management but is now generically used.

The plan sponsor formulates a strategic asset-allocation policy for the total portfolio and implements this investment strategy by selecting various investment managers and allocating assets to each manager. The plan sponsor's decision-making consists of a strategic component represented by the long-term strategic investment policy and an active management portion consisting of tactical asset allocation and selection of active investment managers. Each active investment manager in turn makes a different set of decisions, such as security selection and allocations to sectors, regions, market capitalization, investment style, and so on.

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<sup>1</sup> Bailey, Jeffery V., "Benchmarks and Attribution Analysis for the Total Fund," *Benchmarks and Attribution Analysis*, AIMR Conf. Proc., 2001, no. 3, pp. 28-37.

<sup>2</sup> Campisi, Stephen, "Balanced Portfolio Attribution," *Journal of Performance Measurement*, Winter 2008/2009, pp. 24-35.

Levecq, Christian, "An Exposure-based Attribution Model for Balanced Portfolios," *Journal of Performance Measurement*, Summer 2004, pp. 14-22.

# Introduction (Continued)

## Total Portfolio Versus Micro Attribution

Performance attribution analysis consists of comparing a portfolio's performance with that of a benchmark and decomposing the excess return into pieces to explain the impact of various investment decisions. Since there are two sets of decision-makers in a total portfolio--the plan sponsor and the investment managers--it is important to analyze their contributions separately. The analysis of plan sponsor decisions is referred to as total portfolio attribution, also known as macro attribution or balanced portfolio attribution. The analysis of investment manager decisions is generically called performance attribution, but it is commonly referred to as micro attribution when one wishes to contrast it with total portfolio attribution. This document will focus on total portfolio attribution of plan sponsors' decisions. For more information on micro attribution for equity investment managers, please refer to the Morningstar Equity Performance Attribution Methodology document.

An example of plan sponsor decisions versus investment manager decisions and appropriate use of total portfolio attribution versus micro attribution is illustrated below. This example will be used throughout this document.

**Plan Sponsor Decision**  
Total Portfolio Attribution



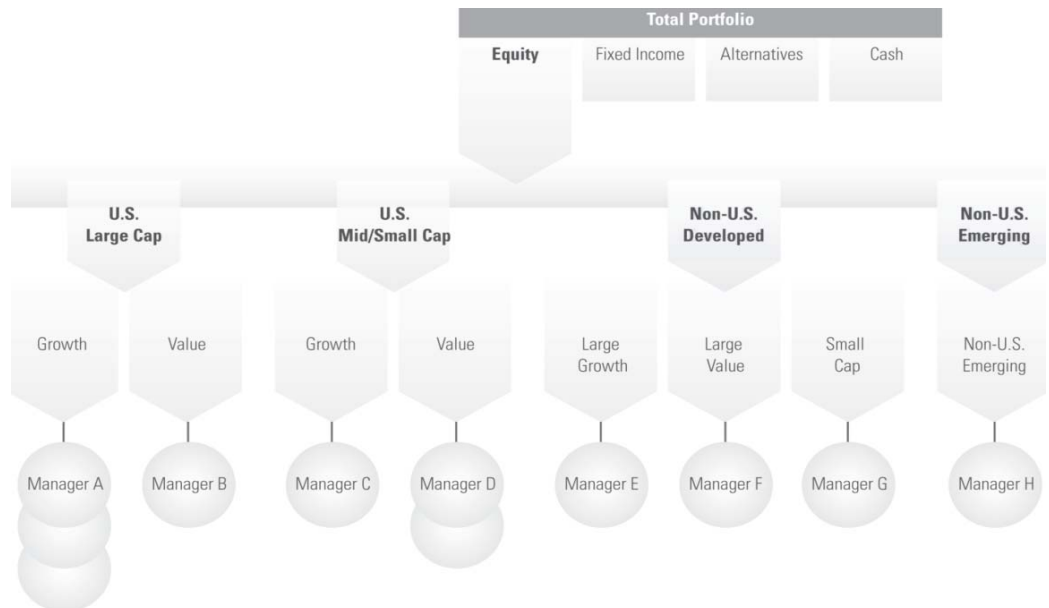
**Manager Decision**  
Micro Attribution

# Introduction (Continued)

## Plan Sponsor Decisions

A plan sponsor's decision is top-down, as the asset-allocation decision is usually made first, and investment manager selection follows as the implementation of the asset-allocation policy. Decision-making is hierarchical and may have multiple layers. For example, an asset allocator may first decide on broad asset class policy weightings such as equity, fixed income, alternatives, and cash. The next policy decision could be a further breakdown into domestic versus foreign, market capitalization, fixed-income quality, types of alternative investments, and so on. This could be followed by investment style-based policy decisions to diversify between growth versus value and various types of fixed income. As the last step in this process, investment managers are selected under each classification.

Below is an example of a hierarchical decision-making process for a U.S.-based plan sponsor:



## Introduction (Continued)

### **Effects Versus Components**

When performing attribution analysis, it is important to distinguish between effects and components, a concept that is also emphasized in micro attribution analysis described in the Morningstar Equity Performance Attribution Methodology document. An effect measures the impact of a particular investment decision. An effect can be broken down into several components that provide insight on each piece of an overall decision, but each piece in isolation cannot represent the plan sponsor's decision. For example, a plan sponsor may make an active decision on asset class weighting by overweighting certain asset classes and underweighting others. Since overweighting certain asset classes necessitates underweighting others and vice versa, the decision is on the entire set of asset class weightings. To better understand the asset class weighting effect, one may examine contributions of individual asset classes. These contributions are simply components that provide additional insight. However, each of these contributions cannot be used in isolation to measure the impact of a decision, as it is not meaningful to say that a plan sponsor made a particular decision to time exposure to the equity broad asset class, for example.

## Introduction (Continued)

### Review of the Classic Approach: Brinson, Hood, and Beebower

Today's approaches to performance attribution are founded on the principles presented in an article<sup>3</sup> written by Brinson, Hood, and Beebower (BHB) published in 1986. Therefore, it is important to review the BHB model even though the model in its original form is not adopted. The study is based on the concept that a portfolio's return consists of the combination of group (for example, asset class) weightings and returns, and decision-making is observed when weightings or returns of the portfolio vary from those of the benchmark. Thus, notional portfolios can be built by combining active or passive group weightings and returns to illustrate the value-add from each decision.

The study deconstructs the value-added return of the portfolio into three parts: tactical asset allocation, stock selection, and interaction. The formulas for these terms are defined below:

Tactical Asset Allocation	=	II - I	=	$\sum_i (w_i^P - w_i^B) \cdot R_i^B$
Stock Selection	=	III - I	=	$\sum_i w_i^B \cdot (R_i^P - R_i^B)$
Interaction	=	IV - III - II + I	=	$\sum_i (w_i^P - w_i^B) \cdot (R_i^P - R_i^B)$
Total Value Added	=	IV - I	=	$\sum_i w_i^P \cdot R_i^P - w_i^B \cdot R_i^B$

These formulas are based on four notional portfolios. These notional portfolios are constructed by combining different weightings and returns, and they are illustrated in the following chart:

		Returns	
		Portfolio	Benchmark
Weights	Portfolio	<b>IV</b> $\sum w_j^P \cdot R_j^P$ Portfolio Return	<b>II</b> $\sum w_j^P \cdot R_j^B$
	Benchmark	<b>III</b> $\sum w_j^B \cdot R_j^P$	<b>I</b> $\sum w_j^B \cdot R_j^B$ Benchmark Return

<sup>3</sup> Brinson, Gary P., Hood, L. Randolph, and Beebower, Gilbert L. "Determinants of Portfolio Performance," *Financial Analysts Journal*, July-August 1986, pp. 39-44.

## Introduction (Continued)

Where:

$w_j^B$	=	The benchmark's weighting for group $j$
$w_j^P$	=	The portfolio's weighting for group $j$
$R_j^B$	=	The benchmark's return for group $j$
$R_j^P$	=	The portfolio's return for group $j$

The tactical asset-allocation effect, also known as the weighting effect, is the difference in returns between notional portfolios II and I. Notional portfolio II represents a hypothetical tactical asset allocator that focuses on how much to allocate to each group (for example, asset class) but purchases index products for lack of opinions on which stocks would perform better than others. Notional portfolio I is the benchmark, which, by definition, has passive group weightings and returns. These two notional portfolios share the same passive group returns but have different weightings; thus, the concept intuitively defines the weighting effect as the result of active weighting decision and passive stock-selection decision.

The stock-selection effect, also known as the selection effect, is the difference in returns between notional portfolios III and I. Notional portfolio III represents a hypothetical security picker that focuses on picking the right securities within each group but mimics how much money the benchmark allocates to each group because the picker is agnostic on which groups would perform better. As described above, notional portfolio I is the benchmark, which has passive group weightings and returns. These two notional portfolios share the same passive group weightings but have different group returns; thus, the concept intuitively defines the selection effect as the result of passive weighting decision and active stock-selection decision.

While the weighting and selection effects are intuitive, the interaction portion is not easily understood. The interaction term, as its name suggests, is the interaction between the weighting and the section effects and does not represent an explicit decision of the investment manager.

The Morningstar methodology for total portfolio performance attribution is founded on the principles of the BHB study, but the BHB model in its original form is not adopted. First, the BHB model is at the total portfolio level and does not break down attribution effects into group-level components. The next section presents the Brinson and Fachler model; this method addresses group-level components. Furthermore, much has evolved in the field of performance attribution since the BHB study. Methodologies are needed to accommodate for multiple hierarchical weighting decisions, perform multiperiod analysis, and so on. These topics are addressed in subsequent sections of this document.



## Introduction (Continued)

### Review of Attribution Components: Brinson and Fachler

The BHB model presented in the previous section shows how attribution effects are calculated. As discussed in the "Effects versus Components" section of this document, an effect can be broken down into several components. Today's approaches to component-level attribution are based on concepts presented in a study<sup>4</sup> by Brinson and Fachler (BF) in 1985. In this article, the impact of a weighting decision for a particular group  $j$  is defined as  $(w_j^P - w_j^B) \cdot (R_j^B - R^B)$ .

The  $(w_j^P - w_j^B)$  portion of this formula is the same as the equation for the tactical asset-allocation effect in the BHB study. It is the difference between the portfolio's weighting in this particular group (for example, asset class) and the benchmark's weighting in the same group, representing the investment manager's weighting decision. In the BHB model, the  $(w_j^P - w_j^B)$  portion is multiplied by the benchmark's total return. This basic principle is preserved in the BF model as the latter also uses the benchmark return. However, in order to gain insight into each group's value-add, the term is transformed into the return differential between the group in question and the total return. Thus, this term intuitively illustrates that a group is good if it outperforms the total. This formula is not in conflict with the BHB model because their results match at the portfolio level--in other words, the sum of BF results from all groups equals the BHB tactical asset-allocation effect.

With the two multiplicative terms of the formula combined, the BF formula illustrates that it is good to overweight a group that has outperformed and underweight a group that has underperformed. This is because overweighting produces a positive number in the first term of the formula, and outperformance yields a positive number in the second term, leading to a positive attribution result. Similarly, a negative weighting differential of an underweighting combined with a negative return differential of an underperformance produces a positive attribution result. Furthermore, it is bad to overweight a group that has underperformed and underweight a group that has outperformed because these combinations produce negative results. This concept is illustrated in the chart below:

	Underperform	Outperform
Overweight	—	+
Underweight	+	—

<sup>4</sup> Brinson, Gary P., and Fachler, Nimrod. "Measuring Non-US Equity Portfolio Performance," *Journal of Portfolio Management*, Spring 1985, pp.73-76.

# Basic Mathematical Expressions

Since attribution formulas use many mathematical expressions in common, these mathematical expressions and their formulas are defined in this section and are used throughout the document.

## Policy and Actual Weightings

The formulas for policy and actual weightings are as follows:

$$[1] \quad w_g^B = \begin{cases} \frac{\text{policy weight of } g}{w_{\emptyset}^B} & \text{if } |g| = N \\ \sum_{h \in \Omega_g} \frac{w_h^B}{w_{\emptyset}^B} & \text{if } |g| < N \end{cases}$$

$$[2] \quad w_g^P = \begin{cases} \frac{\text{actual portfolio weight of investment } g}{w_{\emptyset}^P} & \text{if } |g| = M \\ \sum_{h \in \Omega_g} \frac{w_h^P}{w_{\emptyset}^P} & \text{if } |g| < M \end{cases}$$

Where:

$w_g^B$	=	The policy benchmark's weighting for group $g$
$w_g^P$	=	The actual portfolio weighting for group $g$
$g$	=	The vector that denotes the group
$ g $	=	The number of elements in the vector $g$ , representing the hierarchy level of the group
$N$	=	The level that represents the last hierarchy of the asset-allocation policy
$M$	=	The level that represents the investment level, that is, the last grouping hierarchy
$\Omega_g$	=	All of the subgroups within the group $g$ that are one hierarchy level below
$\emptyset$	=	The total level, which is the total of the portfolio or benchmark

Note:

- ▶ Investment is a generic term for an investment product, such as a mutual fund, separate account, exchange-traded-fund, annuity, security, and so on.
- ▶ All formulas in this document assume that the individual constituents and the results are expressed in decimal format. For example, the number 0.15 represents 15%.
- ▶ Morningstar performs attribution analysis using weightings from the beginning of the period. For example, in conducting attribution analysis for the month of April, returns are based on the period from April 1 to April 30, while weightings are as of the beginning of the month, which is March 31. Refer to Appendix B for display options for weightings.

## Basic Mathematical Expressions (Continued)

A "group" represents a basket of investments classified by the end user. For example, a basket may be an asset class like domestic large-cap equity, foreign equity, domestic investment-grade fixed income, real estate securities, and so on. Group is the most generic term that represents a group of investments or a single investment, in other words, it can be used in referring to an asset class such as foreign equity or a particular investment like a separate account managed with Hansberger Global Equity. The  $g$  symbol represents the vector that denotes the group. In our U.S.-based plan sponsor example illustrated on page 5, equity is the first broad asset class, represented by  $g = (1)$ . Within the equity broad asset class, U.S. mid/small cap is the second asset class, denoted as  $g = (1,2)$  because this is the second asset class within the first broad asset class. Similarly, U.S. mid/small cap growth is referred to as  $g = (1,2,1)$  because it is the first investment style within U.S. mid/small cap; thus, it is the first broad asset class' second asset class' first investment style. Continuing with this example, manager C is denoted as  $g = (1,2,1,1)$  because it is the first manager under U.S. mid/small cap growth. When  $g = \emptyset$ , it represents the null set that denotes the total level such as the total portfolio or the total benchmark.

The  $|g|$  symbol is the number of elements in the vector  $g$ , representing the decision level of the group in the hierarchy. In our example,  $|g| = 1$  stands for the broad asset class level consisting of equity, fixed income, alternatives, and cash, because it is the first level of decision. Note that  $|g| = 0$  represents the total level such as the total benchmark or the total portfolio.  $M$  denotes the level that represents the manager level, that is, the last decision in the hierarchy. In our example,  $M = 4$  because manager selection is the fourth level of decision.

The  $\Omega_g$  expression represents all of the subgroups within the group  $g$  that are one hierarchy level below. Think of a family tree and let each decision level be a generation of relatives: the  $\Omega_g$  symbol represents all of the children of the same parent  $g$ . In our example, the equity broad asset class is denoted by  $g = (1)$ . When using formula [1] to calculate the benchmark weighting of the equity broad asset class, the  $\Omega_{(1)}$  symbol represents all of the subgroups within the equity broad asset class. These are the equity asset classes such as U.S. large cap, U.S. mid/small cap, non-U.S. developed and non-U.S. emerging, which are denoted by  $g = (1,1)$ ,  $g = (1,2)$ ,  $g = (1,3)$ , and  $g = (1,4)$ , correspondingly. Thus, the formula simply states that the policy weighting of the equity broad asset class is the sum of the policy weightings of these four equity asset classes. Interpretation of formula [2] is similar to that of formula [1], but the portfolio's actual weightings are involved instead of policy weightings.

## Basic Mathematical Expressions (Continued)

As stated above, when  $g = \emptyset$ , it represents the null set that denotes the total level such as the total portfolio or the total benchmark. The weightings of the total benchmark and portfolio are included as the denominator in formulas [1] and [2] to rescale the weightings in the extremely rare situation when they do not add up to 100%. Under normal circumstances, these two weightings should be 100%.

### Benchmark and Portfolio Returns

The formulas for benchmark and portfolio returns, expressed in gross of fees, net of fees, and market return formats, are as follows:

$$[3] \quad R_g^B = \begin{cases} \text{gross of fees return on benchmark} & \text{if single benchmark} \\ \frac{\sum_{h \in \Omega_g} w_h^B \cdot R_h^B}{w_g^B} & \text{if blended benchmark} \end{cases}$$

$$[4] \quad R_g^{B'} = \begin{cases} \text{net of fees return on benchmark} & \text{if single benchmark} \\ \frac{\sum_{h \in \Omega_g} w_h^B \cdot R_h^{B'}}{w_g^B} & \text{if blended benchmark} \end{cases}$$

$$[5] \quad R_g^{\tilde{B}} = \begin{cases} \text{market return on benchmark} & \text{if single benchmark} \\ \frac{\sum_{h \in \Omega_g} w_h^B \cdot R_h^{\tilde{B}}}{w_g^B} & \text{if blended benchmark} \end{cases}$$

$$[6] \quad R_g^P = \begin{cases} \text{gross of fees return on investment } g & \text{if } |g| = M \\ \frac{\sum_{h \in \Omega_g} w_h^P \cdot R_h^P}{w_g^P} & \text{if } |g| < M \end{cases}$$

$$[7] \quad R_g^{P'} = \begin{cases} \text{net of fees return on investment } g & \text{if } |g| = M \\ \frac{\sum_{h \in \Omega_g} w_h^P \cdot R_h^{P'}}{w_g^P} & \text{if } |g| < M \end{cases}$$

## Basic Mathematical Expressions (Continued)

$$[8] \quad R_g^{\tilde{P}} = \begin{cases} \text{market return on portfolio} & \text{if } |g| = M \\ \frac{\sum_{h \in \Omega_g} w_h^P \cdot R_h^{\tilde{P}}}{w_g^P} & \text{if } |g| < M \end{cases}$$

Where:

$R_g^B$	=	The benchmark's gross of fees return for group $g$
$R_g^{B'}$	=	The benchmark's net of fees return for group $g$
$R_g^{\tilde{B}}$	=	The benchmark's market return for group $g$
$R_g^P$	=	The portfolio's gross of fees return for group $g$
$R_g^{P'}$	=	The portfolio's net of fees return for group $g$
$R_g^{\tilde{P}}$	=	The portfolio's market return for group $g$

Formulas [3], [4], and [5] refer to benchmark returns for a particular group of investments, and they represent the gross of fees, net of fees, and market return, correspondingly. In most cases benchmark returns are index returns or blends of index returns. Since indexes do not contain fees, their gross and net of fees returns are the same. Some plan sponsors prefer to use investable benchmarks such as index funds or exchange-traded funds where fees exist. When investable indexes are used, their gross and net of fees returns differ, and the distinction between formulas [3] and [4] becomes meaningful. Similarly, formulas [6] and [7] denote gross and net of fees portfolio returns for a particular group of investments, correspondingly. Exchange-traded and closed-end funds have two types of prices, one based on the net asset value and the other based on market price. Formulas [4] and [7] refer to net of fees returns based on NAV, and formulas [5] and [8] refer to market returns based on market price.

## Basic Mathematical Expressions (Continued)

### Note:

- ▶ For separate accounts in the Morningstar database where gross of fees returns are reported, the user-entered annual percentage fee is used in computing net of fees portfolio returns. In such cases,  $R_g^{P'} = (1 + R_g^P) / (1 + f)^{d/365} - 1$ , where  $f$  is the user-entered annual fee (converted into decimal format) and  $d$  is the number of days in the period in question.
- ▶ For U.S. investments in the Morningstar database other than stocks and separate accounts (such as mutual funds, ETFs, and so on), historical annual report net expense ratios are used in computing gross returns. Please refer to the Morningstar Gross Return Methodology document for more details.
- ▶ For non-U.S. investments in the Morningstar database other than stocks and separate accounts where gross of fees returns are not available, the user-entered annual percentage fee is used in computing gross of fees returns. An example of these non-U.S. investments is an open-end mutual fund domiciled outside of the U.S. In such cases,  $R_g^P = (1 + R_g^{P'}) \bullet (1 + f)^{d/365} - 1$ , where  $f$  is the user-entered annual fee (converted into decimal format) and  $d$  is the number of days in the period in question. The fee defaults to the most recent annual report net expense ratio.
- ▶ For position accounts, which are position-based user-defined funds, returns are assumed to be net of fees. Similar to the condition described in the previous bullet point, the user-entered annual percentage fee is used in computing gross of fees returns based on the formula stated above.
- ▶ For U.S. investments other than stocks and separate accounts where gross of fees returns are not available in the Morningstar database, gross of fees returns are assumed to be the same as net of fees returns.
- ▶ In situations other than closed-end and exchange-traded funds where there is no distinction between NAV price and market price, market return is assumed to be the same as net of fees return.
- ▶ When the benchmark is an investable index in the Morningstar U.S. database, such as U.S. exchange-traded funds or mutual funds, benchmark returns are distinguished among gross of fees, net of fees (that is, NAV), and market return whenever such a distinction is available in the Morningstar database. In situations such as a conventional index where only one type of return is available, all three types of returns are assumed to be the same.

## Basic Mathematical Expressions (Continued)

### **Special Situation: Groups Without Weightings**

If neither the portfolio nor the benchmark has a weighting in a particular group, this group should be ignored in order to provide a meaningful attribution analysis.

If the portfolio does not have investments in a particular group but the benchmark does, the group's portfolio weighting is zero, and the group's portfolio return is null. This rule applies regardless of whether the group represents long or short positions. For example, the asset-allocation policy has a target weighting in real estate securities, but the portfolio is not currently invested in the asset class. In this case the active return is attributable entirely to the asset-allocation weighting effect and not subsequent decisions such as manager selection. This makes intuitive sense as the decision to differ from the policy weighting is a weighting effect.

If the portfolio has investments in a particular group but the policy does not have a target weighting in the same group, the group's benchmark return is still the return of the benchmark index that represents this group. For example, the cash asset class may not have a target policy weight; however, the asset class has a benchmark return, which is the return of the representative index.

# Single-Period Methodology

## **Determination of Single Period**

This section addresses the methodology for single-period performance attribution calculations, serving as a foundation for multiperiod analysis presented in the last section of this document. For holdings-based performance attribution analysis, "single period" refers to a buy-and-hold period. Single periods are determined based on the beginning date, ending date, portfolio update date, policy update date, and month-end. Portfolio update date refers to the as-of date when an update of portfolio holdings is provided, due to reallocation, hiring and firing of managers, and so on. Policy update date is the as-of date when the policy is changed, including addition of new asset classes, rebalancing, target allocation change, and so on. When a portfolio or policy update occurs, two single periods are created where the first ends on the day of the update and the second starts on the day after. This method reflects the assumption that activities are executed at the end of the day. For example, if the policy changes on the 10th and the portfolio updates on the 20th, the month is broken into three pieces for calculation purposes, from the 1st to the 10th, 11th to the 20th, and 21st to month-end.

When a portfolio update or a display period requires a mid-month valuation and partial-month returns, investments that are not valued on a daily basis present a challenge. For a separate account or a hedge fund in the Morningstar database, returns are assumed to be zero for the first partial-month periods, and the entire month's gross and net returns are assumed to be earned during the last partial-month period. For example, let us assume that there are two portfolio updates during a particular month, on the 12th and the 26th. The first partial-month period is from the 1st to the 12th, and its gross and net returns are assumed to be zero. The same applies to the period from the 13th to the 26th. The database gross return for the month is assumed to be the gross return for the last single period, from the 27th to the 31st. Similarly, the calculated net return for the month is assumed to be the net return for the last single period.

For a user-defined fund, the underlying holdings (and transactions, where available) are used in determining portfolio valuation and return. If a security in a user-defined fund is not in the Morningstar global equity database, the user-defined price for this security is used in determining the valuation and returns. If such a price is not available on the date in question, the available price prior to this date is carried over. In such cases, returns are calculated based on the change in price; in other words, it is capital appreciation.



## Single-Period Methodology (Continued)

### Strategic Asset-Allocation Policy

The goal of attribution is to break down the excess return of the portfolio over the benchmark into pieces to explain the impact of various investment decisions. As discussed in the Introduction of this document, plan sponsors add value by making several decisions, the first of which is the formulation of the strategic asset-allocation policy. In total portfolio attribution, the impact of a plan sponsor's strategic asset-allocation policy is reflected in the total benchmark return. This benchmark return consists of the combination of policy weightings in each asset class and passive index returns (or investable benchmarks) used as proxies to represent the returns of each asset class. The benchmark return is defined in formulas [3], [4], and [5] in the "Basic Mathematical Expressions" section of this document.

### Weighting and Selection Effects

The weighting effect, also known as the allocation effect, measures the value-add of a plan sponsor's tactical asset-allocation decision. Manager Selection effect reflects the portion of performance attributable to the plan sponsor's skill in selecting managers that outperform their benchmarks. These two effects are described in this because their formulas are similar. Benchmark misfit ("misfit") effect columns may appear following a weighting or selection effect in more advanced attribution analyses, and benchmark misfit attribution is explained in the next section.

The formulas for the weighting and selection effects, expressed in both component and effect formats, are as follows:

$$[9] \quad CA_g = \begin{cases} \left( w_g^P - \frac{w_g^P}{w_{\bar{g}}^B} \cdot w_g^B \right) \cdot (R_g^B - R_{\bar{g}}^B) & \text{if } |g| < M \\ w_g^P \cdot (R_g^P - R_g^B) & \text{if } |g| = M \end{cases}$$

$$[10] \quad EA_{g,n} = \begin{cases} \sum_{h \in \Omega_g} CA_h & \text{if } n = |g| + 1 \\ \sum_{h \in \Omega_g} EA_{h,n} & \text{if } n > |g| + 1 \end{cases}$$

Where:

$CA_g$	=	Component attributable to group $g$ , calculated based on arithmetic method
$EA_{g,n}$	=	Effect attributable to group $g$ at decision level $n$ , based on arithmetic method
$\bar{g}$	=	The group in which group $g$ belongs in the prior grouping hierarchy level

## Single-Period Methodology (Continued)

### Weighting Effect

The weighting effect, also known as allocation effect, measures the value-add of a plan sponsor's tactical asset-allocation decision. Tactical asset allocation refers to the deviation of the portfolio's actual allocation from the long-term policy allocation due to the plan sponsor's bullish and bearish views on different asset classes or the rebalancing policy. Rebalancing is realigning allocation back to policy weightings when the portfolio's allocation naturally drifts from policy as the result of market conditions causing the weightings of higher-returning asset classes to increase and lower-returning asset classes to decrease. Daily rebalancing is impractical due to transaction costs, operational issues, taxes, and other factors. The plan sponsor makes a trade-off decision on when to rebalance, and this is reflected in the tactical asset allocation. When the decision-making process involves multiple hierarchical levels of weighting decisions, such as the example shown in the Introduction, there are multiple weighting effects.

In formula [9], the weighting effect component is the equation associated with the condition  $|g| < M$ . This equation is founded on the Brinson and Fachler (BF) model. To fully understand the equation, let us first focus on its first multiplicative term. The BF model is founded on the concept of the weighting effect being the difference between actual and policy weightings, and the first term of the component formula is essentially that difference. The dissimilarity between the BF model and the component formula stems from the latter being modified for a hierarchical decision-making structure where weighting at each decision level is anchored upon the weighting of the prior decision. For example, let the portfolio's actual weighting in the equity broad asset class be 60% and the policy's weighting be 30%, representing a double weighting. Suppose that, for simplicity, the policy is equally allocated as 7.5% in each of the four equity asset classes: U.S. large cap, U.S. mid/small cap, non-U.S. developed markets and non-U.S. emerging markets. Further assume that the actual weightings are also equally allocated as 15% in each of the four asset classes. Compared with the policy weightings in these four asset classes, these actual weightings of 15% may look overweight compared with 7.5% at first glance. However, careful examination reveals that they are mimicking the policy's allocation in these four asset classes, but their weightings are double only because the equity broad asset class is overweight. Therefore, one must not compare the portfolio actual weighting of U.S. Large Cap directly with the policy weighting in the same asset class. The fair comparison is to create an anchoring system like formula [9] where the policy weighting in the asset class is scaled to the proportion between the actual and the policy weightings in the equity broad asset class. In this example, the policy weighting in U.S. Large Cap must be multiplied by 2 before it can be compared with the policy weighting in the same asset class because 2 is the result of 0.6 divided by 0.3.

## Single-Period Methodology (Continued)

In formula [9] a new symbol,  $\bar{g}$ , is introduced;  $\bar{g}$  is the group in which group  $g$  belongs in the prior decision level of the hierarchy. Following the analogy of a family tree,  $\bar{g}$  represents the parent of  $g$ . For example,  $\bar{g}$  for the U.S. large cap asset class is the equity broad asset class, as U.S. large cap is part of the equity broad asset class. For simplicity, let us call this the "parent group" of group  $g$ . When  $\bar{g} = \emptyset$ , when the parent group is the total level,  $w_{\emptyset}^P = w_{\emptyset}^B = 1 = 100\%$ .

Shifting focus to the second term of the equation associated with the condition when  $|g| < M$  in formula [9], this term is similar to the BF model. In order to adopt a hierarchical structure, the second term is transformed into the return differential between the group in question and its parent group. Thus, this term intuitively illustrates that a group is good if it outperforms the combined performance of all siblings, and vice versa. For example, if U.S. large cap has a benchmark return of 8%, while the equity broad asset class has a benchmark return of 3%, the differential is 5%, a positive number demonstrating that this asset class has outperformed the aggregate of all four asset classes in the equity broad asset class.

In summary, the weighting effect component, the equation associated with the condition when  $|g| < M$  in formula [9], illustrates the same intuitive concepts in the BF article. It is good to overweight a group that has outperformed and underweight a group that has underperformed. It is bad to overweight a group that has underperformed and underweight a group that has outperformed.

Formula [9] shows that the effect of a parent group is the sum of the components or effects of all of its children. The effect of a grandparent group is the sum of the effects of all of its children. Following the same example, the weighting effect of the equity broad asset class is the aggregate of weighting effect figures belonging to U.S. large cap, U.S. mid/small cap, non-U.S. developed markets, and non-U.S. emerging markets. Similarly, the weighting effect of the U.S. large cap asset class is the sum of the effect numbers from the U.S. large cap growth and U.S. large cap value investment styles.

## Single-Period Methodology (Continued)

### Manager Selection

The manager selection effect reflects the portion of performance attributable to the plan sponsor's skill in selecting managers that outperform their benchmarks. In formula [9], this is the equation associated with the condition  $|g| = M$ . This equation is very intuitive, as it is the investment manager's excess return over the benchmark multiplied by the actual weighting in the manager. This is the transition point between total portfolio attribution and micro attribution, as the manager's excess return can be further broken down into weighting and selection effects using micro attribution. For computation of the manager selection effect, gross of fees returns are used wherever available, as they are consistent with micro attribution's active return (that is, excess return). Micro attribution's active return is the weighted average return of a portfolio's underlying securities minus the weighted average return of the benchmark's underlying securities, both of which do not inherently take fees into consideration.

Formula [10] is used in aggregating manager selection effect component numbers into manager selection effect figures for higher hierarchical levels. Applying the parent-children analogy, this equation simply states that a parent's manager selection effect is the sum of the manager selection figures of all of its children. Following the same example shown in the Introduction, the U.S. large cap asset class' manager selection effect is the sum of the manager selection effect components of Manager A and Manager B, the two managers belonging to this asset class.

## Single-Period Methodology (Continued)

### Benchmark Misfit

Benchmark misfit effect columns may appear following a weighting or selection effect in more advanced attribution analyses. They are named after the column that precedes them, for example, asset class misfit, investment style misfit, manager misfit, and so forth. The concept of Benchmark Misfit is presented in an article written by Jeffery V. Bailey.<sup>5</sup> It is the difference between the return of a higher hierarchical level benchmark and the aggregate return on the indexes of a lower hierarchical level. For example, the S&P 500 index is chosen to represent the equity broad asset class, and various MSCI and Russell indexes are selected for the domestic and foreign equity asset classes. In this case, the weighted sum of the returns of the MSCI and Russell indexes does not equal to the return of the S&P 500 index. The difference does not represent a plan sponsor's skill but rather is a reflection of a benchmark misfit; therefore, it should be isolated from the decision-based weighting effect. Benchmark misfit is zero when the return of a higher hierarchical level benchmark is the combination of policy weightings and returns of indexes at a lower hierarchical level. This type of benchmark is also known as blended benchmark.

The formula for benchmark misfit is as follows:

$$[11] \quad MA_{g,n} = \begin{cases} \left( \frac{w_g^P}{w_g^B} \bullet w_g^B \right) \bullet (R_g^B - R_{\bar{g}}^B) & \text{if } |g| < M \\ w_g^P \bullet (R_g^B - R_{\bar{g}}^B) & \text{if } |g| = M \\ \sum_{h \in \Omega_g} MA_{h,n} & \text{if } n \geq |g| + 1 \end{cases}$$

Where:

---

$MA_{g,n}$  = Benchmark misfit that is attributable to group  $g$  at decision level  $n$ , based on arithmetic method

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<sup>5</sup> Bailey, Jeffery V., "Benchmarks and Attribution Analysis for the Total Fund," Benchmarks and Attribution Analysis, AIMR Conf. Proc., 2001, no. 3, pp. 28-37.

## Single-Period Methodology (Continued)

The benchmark misfit is expressed as three equations in formula [11]. The condition  $|g| < M$  refers to the benchmark misfit at hierarchical levels above the investment manager level, such as asset class misfit and investment style misfit in the example presented in the Introduction of this document. In such cases, the misfit is defined as the anchored policy weighting multiplied by the difference between the returns of the benchmark for this particular group and that of the prior hierarchy. The concept of anchoring is discussed in the "Weighting Effect" section above. For example, let the non-U.S. developed small cap investment style's benchmark be the MSCI EAFE Small Cap Index and the benchmark for the non-U.S. developed asset class the MSCI EAFE Index. The difference in returns between these two indexes multiplied by the anchored policy weighting of the non-U.S. developed small cap investment style produces the benchmark misfit associated with this investment style.

The condition  $|g| = M$  describes the benchmark misfit at the investment manager level, the lowest level of total portfolio attribution analysis. The equation intuitively demonstrates that the benchmark misfit is the actual weighting multiplied by the difference between the returns of the manager's benchmark and the benchmark for the prior hierarchy. For example, a non-U.S. small cap value manager may be compared with the MSCI EAFE Small Value Index, while the investment style uses a more general index such as the MSCI EAFE Small Cap Index. This manager's benchmark misfit is the actual portfolio weighting in the manager multiplied by the difference between the return of the MSCI EAFE Small Value Index and that of the MSCI EAFE Small Cap Index.

Following the parent-children analogy, the  $n \geq |g| + 1$  condition in formula [11] simply states that a parent's benchmark misfit is the sum of the benchmark misfit figures of all of its children. For example, when analyzing the investment style benchmark misfit, use the  $|g| < M$  condition to calculate the benchmark misfit statistics of U.S. large cap growth and U.S. large cap value investment styles and use the  $n \geq |g| + 1$  condition to add these two misfit figures to form the investment style benchmark misfit measure for the U.S. large cap asset class.

## Single-Period Methodology (Continued)

### Manager Fee

The manager fee effect represents the impact of investment manager fees on the return of the investment; it is the weighted differential between an investment's gross of fees and net of fees returns. Unlike weighting and selection effects, the manager fee effect does not represent an explicit decision of the plan sponsor. However, it is included in an attribution analysis because it is an important component of the performance of an investment manager, as an active manager's mandate is to outperform the designated benchmark after accounting for fees. Since index returns do not contain fees, some plan sponsors prefer to use investable benchmarks such as index funds or exchange-traded funds. When investable indexes are used, the manager fee effect shows the fee differential compared with the investable index.

The manager fee effect is defined as follows:

$$[12] \quad FA_g = \begin{cases} w_g^P \cdot (R_g^{P'} - R_g^P) - w_g^B \cdot (R_g^{B'} - R_g^B) & \text{if } |g| < M \\ w_g^P \cdot [(R_g^{P'} - R_g^P) - (R_g^{B'} - R_g^B)] & \text{if } |g| = M \end{cases}$$

Where:

---

$$FA_g = \text{Manager fee attribution result that is attributable to group } g, \text{ based on arithmetic method}$$

---

The manager fee effect is expressed as two equations in formula [12], to be selectively applied depending on the situation. The condition  $|g| < M$  refers to the manager fee effect at hierarchical levels above the investment manager level, such as at the asset class level. The first part of the equation is the portfolio's weighted fee, and it is the differential between the portfolio's net and gross returns weighted by the portfolio's actual weighting in the asset class. The second part of the equation is the benchmark's weighted fee, and it is the differential between the benchmark's net and gross returns weighted by the policy weighting in the asset class. This number is zero for a conventional benchmark and is not zero when investable indexes are involved. The manager fee effect is the difference between the portfolio's weighted fee and that of the benchmark.

The condition  $|g| = M$  refers to the manager fee effect at the manager level. The formula is the same in spirit but is expressed differently because policy weighting does not exist at the manager level and must be substituted by portfolio's actual weighting when calculating the benchmark's weighted fee.

## Single-Period Methodology (Continued)

### Premium/Discount

Closed-end and exchange-traded funds have two types of prices, one based on the net asset value and the other based on market price. The difference between these two prices is referred to as "premium" when the market price is above the NAV and "discount" when it is below. The premium or discount widens or narrows over time. The return differential caused by a change in premium/discount is not insignificant. When attribution is performed on returns calculated based on NAVs, it reflects the contribution from fund managers but does not represent the economic value of the underlying investment. On the other hand, when attribution is performed on returns computed based on market prices, this correctly reflects the economic value of the underlying investment, but the premium/discount that is outside of the manager's control is incorrectly attributed to a contribution by the manager.<sup>6</sup> The solution is to isolate the return attribution to a change in premium/discount from the manager selection effect. The premium/discount effect is positive when the premium has widened or the discount has narrowed, and it is negative when the premium has shrunk or the discount has grown.

The formula for premium/discount effect is as follows:

$$[13] \quad PA_g = \begin{cases} w_g^P \cdot (R_g^{\tilde{P}} - R_g^{P'}) - w_g^B \cdot (R_g^{\tilde{B}} - R_g^{B'}) & \text{if } |g| < M \\ w_g^P \cdot [(R_g^{\tilde{P}} - R_g^{P'}) - (R_g^{\tilde{B}} - R_g^{B'})] & \text{if } |g| = M \end{cases}$$

Where:

---

$PA_g$  = Premium/discount attribution result that is attributable to group  $g$ , based on arithmetic method

---

Note:

- The premium/discount effect is not calculated when the portfolio does not have any closed-end or exchange-traded fund, regardless of whether they are used as benchmarks.

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<sup>6</sup> Tsai, Cindy Sin-Yi, "Identifying A Sponsor's Impact on Total Returns: Performance Attribution for the Total Portfolio," (white paper)

[http://www.cfainstitute.org/learning/products/publications/contributed/Pages/identifying\\_a\\_sponsor\\_s\\_impact\\_on\\_total\\_returns\\_\\_performance\\_attribution\\_for\\_the\\_total\\_portfolio.aspx](http://www.cfainstitute.org/learning/products/publications/contributed/Pages/identifying_a_sponsor_s_impact_on_total_returns__performance_attribution_for_the_total_portfolio.aspx).



## Single-Period Methodology (Continued)

### Active Return

Active return, which is the total portfolio's excess return over the policy benchmark, is the combination of weighting, manager selection, benchmark misfit (wherever available), manager fee (wherever available), and premium/discount (wherever available) effects. This is defined in formula [14]:

[14]

$$AA_{\emptyset,T,Cum} = R_{\emptyset,T,Cum}^{\tilde{P}} - R_{\emptyset,T,Cum}^{\tilde{B}} = \sum_{n=1}^M EA_{\emptyset,n,T,Cum} + \sum_{n=2}^M MA_{\emptyset,n,T,Cum} + FA_{\emptyset,T,Cum} + PA_{\emptyset,T,Cum}$$

Where:

---

$$AA_{\emptyset} = \text{Portfolio's active return, calculated based on arithmetic method}$$

---

The active return in formula [14] is net of investment manager fees but gross of consultant fees or any expenses born at the plan level. The total portfolio's return net of all fees is computed by deducting the investment consultant fee from the active return. These two terms are expressed in formulas [22] and [23] in the "Multiple-Period Methodology" section of this document.

### Interaction Term of Brinson Models

The classic approaches to performance attribution described above, based on articles written by Brinson, Hood, and Beebower (1986) and Brinson and Fachler (1985), defines three terms for attribution analysis: tactical asset allocation, stock selection, and interaction term. The tactical asset allocation is the weighting effect in Morningstar's Total Portfolio Attribution methodology. The stock or security selection is the manager effect. The interaction term, however, is not explicitly defined in the methodology.

Asset-allocation portfolios follow a top-down decision-making process, and it is common practice for a top-down attribution analysis to embed the interaction term in the selection effect. By embedding, the term is pushed down to the manager's micro attribution level. In other words, the appropriate venue in which the interaction term can be isolated is when a micro attribution analysis is run for the manager to break down this manager's value-add on allocation and security-selection decisions.

# Multiple-Period Methodology

## Overview

The previous section demonstrates how attribution statistics are calculated for each single holding period. This section addresses multiple-period analysis, as it is often desirable to perform an analysis that spans over several portfolio holdings dates, for example, from Jan. 1, 2009, to Sept. 30, 2009. Although one might think of treating this as a single period, that is, taking the portfolio and policy weightings as of Dec. 31, 2008, and applying them to returns from Jan. 1, 2009, to Sept. 30, 2009, valuable information could be lost. Portfolio and policy benchmark constituents and their weightings might have changed between Dec. 31, 2008, and Sept. 30, 2009, due to the addition of new asset classes, hiring and firing of managers, reallocation, rebalancing, and so on.

When changes happen, the month is broken into several single periods in order to capture the change. For example, if the policy changes on the 10th and the portfolio updates on the 20th, the month is broken into three pieces for calculation purpose, from the 1st to the 10th, 11th to the 20th, and 21st to month-end. Returns of these three single periods are compounded, and attribution results are calculated on these three single periods separately before being linked together for reporting. This section demonstrates how these single-period attribution results are accumulated into an overall multiperiod outcome.

The multiple-period methodology described in this document is commonly referred to as arithmetic method. When referring to an arithmetic method, the word "accumulate" is a more appropriate term than the word "link" as components and effects are added over time rather than geometrically compounded. Several arithmetic methodologies have emerged in the industry. These alternative methodologies are commonly referred to as triple-sum, as the cumulative active return (excess return over benchmark) over multiple periods is the sum of components in all groups (for example, asset class), decisions (weighting versus selection), and time periods. Since adding components over time does not equal the cumulative active return, additional mathematical "smoothing" is applied to make them match. Mathematical smoothing is where formulas and philosophies differ among various alternative methodologies. The choice of the method matters.<sup>6</sup> It is important to make sure the choice of method does not significantly distort the reality such as altering the relative results of components and effects or causing a detractor to appear as a contributor or vice versa.

## Multiple-Period Methodology (Continued)

This document presents a modified version of one of the arithmetic methodologies, the Modified Frongello methodology.<sup>7</sup> Morningstar modified the methodology to accommodate for multiple hierarchical levels of actual allocation benchmark, discussed below.

### Notional Portfolio: Actual Allocation Benchmark

Actual allocation benchmark return is the return of a hypothetical portfolio comprised of actual portfolio weightings and gross of fees benchmark return. It is based on the same concept as the notional portfolio II in the Brinson, Hood, and Beebower article, representing a tactical asset allocator that focuses on how much to allocate to each group (for example, asset class) but purchases index products for lack of opinions on which managers would perform better than others. There is a hypothetical portfolio for each hierarchical level of decision, and comparing the returns of these hypothetical portfolios demonstrates the plan sponsor's value-add at each step of the decision process. They also aid formulas [16] and [18] in accumulating single-period attribution results into an overall multiperiod outcome.

The actual allocation benchmark return is defined as follows:

$$[15] \quad R_{\emptyset}^{HL} = \begin{cases} R_{\emptyset}^B & \text{if } L = 0 \\ \frac{\sum_{i=1}^{Q^L} w_i^{P,L} \cdot R_i^{B,L}}{w_{\emptyset}^P} & \text{if } 0 < L < M \\ R_{\emptyset}^P & \text{if } L = M \end{cases}$$

Where:

$R_{\emptyset}^{HL}$	=	Actual allocation benchmark return, calculated based on actual weightings and benchmark returns at level $L$
$Q^L$	=	Number of groups at level $L$
$w_i^{P,L}$	=	Actual portfolio weighting for group $i$ at level $L$
$R_i^{B,L}$	=	Benchmark's gross of fees return for group $i$ at level $L$

<sup>7</sup> Frongello, Andrew Scott Bay, "Readers' Reflections," *Journal of Performance Measurement*, Winter 2002/2003, pp. 7-11. This methodology is a modified version of the authors original work in "Linking Single Period Attribution Results," *The Journal of Performance Measurement*, Spring 2002, pp.10-22.

## Multiple-Period Methodology (Continued)

The actual allocation benchmark is expressed as three equations in formula [15]. The condition  $L = 0$  describes the actual allocation benchmark at the strategic level, and its return is the gross of fees total benchmark return consisting of the combination of policy weightings and gross of fees benchmark returns. The condition  $0 < L < M$  represents the actual allocation benchmark at tactical levels. Following the same example, these levels are broad asset class, asset class and investment style. Actual allocation benchmark return for a particular level is the combination of actual portfolio weightings and gross of fees benchmark returns at the level in question. The condition  $L = M$  refers to the actual allocation benchmark at the manager level, the last level of decision of a plan sponsor's decision tree. Its return is the portfolio's actual return consisting of the combination of the actual portfolio weighting and actual gross of fees return.

Following the example presented in the Introduction of this document, we define the various levels of hierarchy:

- ▶  $L = 0$  refers to the actual allocation benchmark at the strategic level where the benchmark is the gross of fees total benchmark return, calculated as the combination of policy weightings and gross of fees benchmark returns of the four broad asset classes: equity, fixed income, alternatives, and cash.
- ▶  $L = 1$  refers to the actual allocation benchmark at the broad asset class level, computed as the combination of actual portfolio weightings and gross of fees benchmark returns of the four broad asset classes: equity, fixed income, alternatives, and cash.
- ▶  $L = 2$  refers to the actual allocation benchmark at the asset class level, calculated by combining actual portfolio weightings and gross of fees benchmark returns of the four equity asset classes (for example, U.S. large cap) and various fixed income, alternatives, and cash asset classes not illustrated in the example in the Introduction of this document.
- ▶  $L = 3$  refers to the actual allocation benchmark at the investment style level, computed as the combination of actual portfolio weightings and gross of fees benchmark returns of the seven equity investment styles (for instance, U.S. large cap growth) and various fixed income, alternatives, and cash investment styles not illustrated in the example.
- ▶  $L = M = 4$  refers to the actual allocation benchmark at the manager level, and its return is the portfolio's actual return, computed as the combination of actual portfolio weighting and actual gross of fees return of each manager in the portfolio.

## Multiple-Period Methodology (Continued)

Following the example presented in the Introduction, these are the hierarchical levels that need to be plugged into equations [16] and [18]:

- ▶ When calculating the multiperiod broad asset class allocation effect to measure the impact of tactical asset-allocation decision for equity, fixed income, alternatives and cash, use actual allocation benchmarks at  $L = 0$  and  $L = 1$ .
- ▶ When calculating the multiperiod asset class allocation effect and its benchmark misfit to measure the impact of tactical decision for the four equity asset classes (for example, U.S. large cap) and various fixed income, alternatives, and cash asset classes not illustrated in the example in the Introduction of this document, use actual allocation benchmarks at  $L = 1$  and  $L = 2$ .
- ▶ When calculating the multiperiod investment style allocation effect and its benchmark misfit to measure the impact of tactical decision for the seven equity investment styles (for instance, U.S. large cap growth) and various fixed income, alternatives, and cash investment styles not illustrated in the example, use actual allocation benchmarks at  $L = 2$  and  $L = 3$ .
- ▶ When calculating multiperiod manager selection effect and its benchmark misfit to measure the value-add of picking a good manager, use actual allocation benchmarks at  $L = 3$  and  $L = 4$ .

## Multiple-Period Methodology (Continued)

### Formulas for Cumulative Attribution Results

Use the following formulas to accumulate single-period attribution results into multiperiod statistics:

[16]

$$CA_{g,T,Cum} = \frac{(2 + R_{\emptyset,T}^{H|\bar{g}|} + R_{\emptyset,T}^{H|g|})}{2} \bullet CA_{g,T-1,Cum} + \frac{(2 + R_{\emptyset,T-1,Cum}^{H|\bar{g}|} + R_{\emptyset,T-1,Cum}^{H|g|})}{2} \bullet CA_{g,T}$$

[17]

$$EA_{g,n,T,Cum} = \begin{cases} \sum_{h \in \Omega_g} CA_{h,T,Cum} & \text{if } n = |g| + 1 \\ \sum_{h \in \Omega_g} EA_{h,n,T,Cum} & \text{if } n > |g| + 1 \end{cases}$$

[18]

$$MA_{g,n,T,Cum} = \begin{cases} \frac{(2 + R_{\emptyset,T}^{H|\bar{g}|} + R_{\emptyset,T}^{H|g|})}{2} \bullet MA_{g,n,T-1,Cum} + \frac{(2 + R_{\emptyset,T-1,Cum}^{H|\bar{g}|} + R_{\emptyset,T-1,Cum}^{H|g|})}{2} \bullet MA_{g,n,T} & \text{if } n = |g| \\ \sum_{h \in \Omega_g} MA_{h,n,T,Cum} & \text{if } n > |g| \end{cases}$$

$$[19] \quad FA_{g,T,Cum} = \sum_{t=1}^T FA_{g,t} \bullet \frac{(R_{\emptyset,T,Cum}^{P'} - R_{\emptyset,T,Cum}^P) - (R_{\emptyset,T,Cum}^{B'} - R_{\emptyset,T,Cum}^B)}{\sum_{t=1}^T FA_{\emptyset,t}}$$

$$[20] \quad PA_{g,T,Cum} = \sum_{t=1}^T PA_{g,t} \bullet \frac{(R_{\emptyset,T,Cum}^{\tilde{P}} - R_{\emptyset,T,Cum}^{P'}) - (R_{\emptyset,T,Cum}^{\tilde{B}} - R_{\emptyset,T,Cum}^{B'})}{\sum_{t=1}^T PA_{\emptyset,t}}$$

[21]

$$AA_{\emptyset,T,Cum} = R_{\emptyset,T,Cum}^{\tilde{P}} - R_{\emptyset,T,Cum}^{\tilde{B}} = \sum_{n=1}^M EA_{\emptyset,n,T,Cum} + \sum_{n=2}^M MA_{\emptyset,n,T,Cum} + FA_{\emptyset,T,Cum} + PA_{\emptyset,T,Cum}$$

## Multiple-Period Methodology (Continued)

Where:

$CA_{g,T,Cum}$	=	Cumulative component that is attributable to group $g$ , calculated based on arithmetic method, cumulative from single holding periods $1$ to $T$
$EA_{g,n,T,Cum}$	=	Cumulative effect that is attributable to group $g$ at decision level $n$ , calculated based on arithmetic method, cumulative from single holding periods $1$ to $T$
$MA_{g,n,T,Cum}$	=	Cumulative benchmark misfit that is attributable to group $g$ at decision level $n$ , calculated based on arithmetic method, cumulative from single holding periods $1$ to $T$
$FA_{g,T,Cum}$	=	Cumulative manager fee attribution result that is attributable to group $g$ , calculated based on arithmetic method, cumulative from single holding periods $1$ to $T$
$PA_{g,T,Cum}$	=	Cumulative premium/discount attribution result that is attributable to group $g$ , based on arithmetic method, cumulative from single holding periods $1$ to $T$
$AA_{\emptyset,T,Cum}$	=	The portfolio's cumulative active return, based on arithmetic method, cumulative from single periods $1$ to $T$
$R_{\emptyset,T}^{H g }$	=	Actual allocation benchmark return at the level $ g $ , at single holding period $T$
$R_{\emptyset,T,Cum}^B$	=	The benchmark's gross of manager fees return for the total level, cumulative from periods $1$ to $T$
$R_{\emptyset,T,Cum}^P$	=	The portfolio's gross of manager fees return for the total level, cumulative from periods $1$ to $T$
$R_{\emptyset,T,Cum}^{B'}$	=	The benchmark's net of manager fees return for the total level, cumulative from periods $1$ to $T$
$R_{\emptyset,T,Cum}^{P'}$	=	The portfolio's net of manager fees return for the total level, cumulative from periods $1$ to $T$
$R_{\emptyset,T,Cum}^{B''}$	=	The benchmark's market return net of manager fees, cumulative from periods $1$ to $T$
$R_{\emptyset,T,Cum}^{P''}$	=	The portfolio's market return net of manager fees, cumulative from periods $1$ to $T$
$R_{\emptyset,T}^{B'}$	=	The benchmark's net of manager fees return for the total level, at single holding period $T$
$R_{\emptyset,T}^{P'}$	=	The portfolio's net of manager fees return for the total level, at single holding period $T$
$EA_{g,n,T}$	=	Effect at single holding period $T$ for group $g$ decision level $n$ , based on arithmetic method
$MA_{g,n,T}$	=	Benchmark misfit at single holding period $T$ for group $g$ decision level $n$ , based on arithmetic
$FA_{g,T}$	=	Manager fee attribution result at single holding period $T$ for group $g$ , based on arithmetic method

Note:

- ▶ At period  $T=1$ ,  $CA_{g,T,Cum} = CA_g$ ,  $EA_{g,n,T,Cum} = EA_{g,n}$ ,  $MA_{g,n,T,Cum} = MA_{g,n}$ , and  $FA_{g,T,Cum} = FA_g$ , and these terms are defined in the "Single-Period Methodology" section.

## Multiple-Period Methodology (Continued)

$$[22] \quad CF_{T,Cum} = - \left[ (1 + CF_{Ann})^{\frac{d}{365}} - 1 \right]$$

$$[23] \quad NF_{T,Cum} = R_{\emptyset,T,Cum}^{\tilde{P}} + CF_{T,Cum}$$

Where:

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$CF_{T,Cum}$	=	Consultant fee or expenses born at plan level, prorated for the cumulative period
$NF_{T,Cum}$	=	The portfolio's return net of consultant and manager fees, cumulative from periods 1 to $T$
$CF_{Ann}$	=	User-entered annual consulting fee or expenses born at plan level
$d$	=	Number of days in the cumulative period

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Note:

- It is optional for the user to provide the plan-level consultant fee or expenses defined in formulas [22] and [30]. Users have the option to itemize and customize multiple line items of expenses born at the plan level.



## Multiple-Period Methodology (Continued)

### Formulas for Annualized Attribution Results

Use the following formulas to annualize cumulative multiperiod attribution results:

$$[24] \quad CA_{g,T,Ann} = CA_{g,T,Cum} \cdot \frac{y}{m}$$

$$[25] \quad EA_{g,n,T,Ann} = EA_{g,n,T,Cum} \cdot \frac{y}{m}$$

$$[26] \quad MA_{g,n,T,Ann} = MA_{g,n,T,Cum} \cdot \frac{y}{m}$$

$$[27] \quad FA_{g,T,Ann} = FA_{g,T,Cum} \cdot \frac{y}{m}$$

$$[28] \quad PA_{g,T,Ann} = PA_{g,T,Cum} \cdot \frac{y}{m}$$

$$[29] \quad AA_{\emptyset,T,Ann} = AA_{\emptyset,T,Cum} \cdot \frac{y}{m}$$

$$[30] \quad CF_{T,Ann} = -CF_{Ann}$$

$$[31] \quad NF_{T,Ann} = R_{\emptyset,T,Ann}^{\tilde{p}} - CF_{T,Ann}$$

## Multiple-Period Methodology (Continued)

Where:

$CA_{g,T,Ann}$	=	Annualized component that is attributable to group $g$ , calculated based on arithmetic method, over the time period from $1$ to $T$
$EA_{g,n,T,Ann}$	=	Annualized effect that is attributable to group $g$ at decision level $n$ , calculated based on arithmetic method, over the time period from $1$ to $T$
$MA_{g,n,T,Ann}$	=	Annualized benchmark misfit that is attributable to group $g$ at decision level $n$ , calculated based on arithmetic method, over the time period from $1$ to $T$
$FA_{g,T,Ann}$	=	Annualized manager fee attribution result that is attributable to group $g$ , calculated based on arithmetic method, over the time period from $1$ to $T$
$PA_{g,T,Ann}$	=	Annualized premium/discount attribution result that is attributable to group $g$ , calculated based on arithmetic method, over the time period from $1$ to $T$
$AA_{O,T,Ann}$	=	The portfolio's annualized active return, calculated based on arithmetic method, over the time period from $1$ to $T$
$y$	=	The number of periods in a year; for example, it is 12 when data are in monthly frequency
$m$	=	The total number of periods; for example, it is 40 when the entire time period spans over 40 months

Note:

- Annualization methodology of  $(y/m)$  is adopted with the end goal of preserving the arithmetic method's additive property across groups and types of attribution effects.

# Appendix A: Contribution

## Overview

Contribution is a topic that is often associated with attribution. Contribution is an absolute analysis that multiplies the absolute weighting in a sector or security by the absolute return, so an investment with mediocre performance may have a large contribution simply because a large amount of money is invested in it. Attribution, on the other hand, shows relative weighting and relative returns. A good result can come only from overweighting an outperforming investment or underweighting an underperforming one; therefore, it is a better measure of an investment manager's skill. Since contribution is an absolute analysis and attribution is a relative analysis, they complement each other.

## Single Period

In a single-period analysis, the contribution is defined as:

$$[32] \quad C_g^P = \begin{cases} w_g^P \bullet R_g^{P'} & \text{if } |g| = M \\ \sum_{h \in \Omega_g} C_h^P & \text{if } |g| < M \end{cases}$$

$$[33] \quad C_g^B = \begin{cases} w_g^B \bullet R_g^{B'} & \text{if } |g| = M \\ \sum_{h \in \Omega_g} C_h^B & \text{if } |g| < M \end{cases}$$

Where:

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$C_g^P$	=	Contribution toward the portfolio that is associated with group $g$
$C_g^B$	=	Contribution toward the benchmark that is associated with group $g$
$M$	=	The level that represents the security level, that is, the last grouping hierarchy

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Note:

- These formulas demonstrate how to calculate contribution using net of fees return. The same formulas are applicable for market return in the case of exchange-traded and closed-end funds where the market return based on market price differs from the net of fees return based on NAV. This can be accomplished by replacing the latter with the former.

## Appendix A: Contribution (Continued)

### Multiple-Period

Use the following formulas to link single-period contribution into multiperiod results:

$$[34] \quad C_{g,T,Cum}^P = \sum_{t=1}^T (1 + R_{\emptyset,t-1,Cum}^{P'}) \cdot W_{g,t}^P \cdot R_{g,t}^{P'} = \sum_{t=1}^T (1 + R_{\emptyset,t-1,Cum}^{P'}) \cdot C_{g,t}^P$$

$$[35] \quad C_{g,T,Cum}^B = \sum_{t=1}^T (1 + R_{\emptyset,t-1,Cum}^{B'}) \cdot W_{g,t}^B \cdot R_{g,t}^{B'} = \sum_{t=1}^T (1 + R_{\emptyset,t-1,Cum}^{B'}) \cdot C_{g,t}^B$$

$$[36] \quad C_{g,T,Ann}^P = (1 + C_{g,T,Cum}^P)^{\frac{y}{m}} - 1$$

$$[37] \quad C_{g,T,Ann}^B = (1 + C_{g,T,Cum}^B)^{\frac{y}{m}} - 1$$

Where:

$C_{g,T,Cum}^P$	=	Cumulative contribution toward the portfolio that is associated with group $g$ , cumulative from single holding periods $1$ to $T$
$C_{g,T,Cum}^B$	=	Cumulative contribution toward the benchmark that is associated with group $g$ , cumulative from single holding periods $1$ to $T$
$C_{g,T,Ann}^P$	=	Annualized contribution toward the portfolio that is associated with group $g$ , cumulative from single holding periods $1$ to $T$
$C_{g,T,Ann}^B$	=	Annualized contribution toward the portfolio that is associated with group $g$ , cumulative from single holding periods $1$ to $T$
$R_{\emptyset,t-1,Cum}^{P'}$	=	The portfolio's net of fees return for the total level, cumulative from periods $1$ to $t-1$
$R_{\emptyset,t-1,Cum}^{B'}$	=	The benchmark's net of fees return for the total level, cumulative from periods $1$ to $t-1$

At segment levels, such as a particular asset class or manager, the multiperiod contribution is not the geometric compounding of single-period contribution figures. The weightings change may be due to transactions that cause the capital base for the segment to change. Therefore, at the beginning of each period it is necessary to compute the segment's capital base by multiplying the total portfolio's wealth by the segment's weighting before applying the base to the period's contribution. The total portfolio's wealth at the beginning of the period is represented by 1 plus the cumulative portfolio return up to that time, in other words, it is the growth of one dollar. Expressed another way, a segment's multiperiod contribution is the sum of this segment's dollar contributions from every period, assuming a \$1 initial investment in the total portfolio. When contributions are expressed in cumulative terms, segment contributions sum to that of the total portfolio. However, once annualized, these numbers no longer add up.

## Appendix B: Average Weightings

Morningstar provides three options for the display of portfolio actual and policy weightings in attribution analysis: beginning of the period, end of the period, and average. The default is the end of the period. These options are for display purpose only and do not affect the actual calculation of attribution results. Performance measurement and analysis always use beginning of the period weighting for skill evaluations because the beginning of the period is when decisions are made, even though the success of these decisions are evaluated at the end of the period. Therefore, Morningstar attribution as a form of performance analysis uses beginning of the period weightings when calculating results.

This appendix explains the average weighting methodology in details when the average weighting is chosen for display. In a multiperiod analysis, the average weighting of a group is the day-weighted average of the group's weighting at the beginning of analysis period and at every update. This method gives more importance to periods that include more number of days.

The formulas for averaging weightings are as follows:

$$[38] \quad \bar{w}_g^P = \frac{\sum_{t=1}^N d_t w_{g,t}^P}{T}$$

$$[39] \quad \bar{w}_g^B = \frac{\sum_{t=1}^N d_t w_{g,t}^B}{T}$$

Where:

$\bar{w}_g^P$	=	average weighting of group $g$ in the portfolio
$\bar{w}_g^B$	=	average weighting of group $g$ in the policy
$N$	=	number of periods in the multiperiod attribution analysis
$d_t$	=	number of calendar days in period $t$
$w_{g,t}^P$	=	portfolio weighting of group $g$ in period $t$
$T$	=	total number of calendar days in the attribution analysis
$w_{g,t}^B$	=	policy weighting of group $g$ in period $t$

Note:

- ▶ When a manager or asset class is not held during a particular period, its weighting is considered to be zero in the period.

## Appendix B: Average Weightings (Continued)

For example, let's assume that the attribution analysis spans from Jan. 1, 2009, to Dec. 31, 2009, for a portfolio that was initially funded in 1995. During the calendar year 2009, the portfolio had reallocations on April 14, 2009, and Dec. 18, 2009. The calendar year has 365 days. Please refer to the table below for prorated weightings for each set of allocations. The portfolio allocation as of Dec. 31, 2008, is taken into consideration even though the portfolio does not have an update on that date because it is the beginning allocation of the attribution analysis. When computing the average weighting, this allocation is assigned a prorated weighting of 104/365 for being in the portfolio for 104 days out of 365, from Jan. 1, 2009, to April 14, 2009. The April 14, 2009, reallocation percentages are given a prorated weighting of 248/365 for being in the portfolio for 248 days from April 15, 2009, to Dec. 18, 2009. Finally, the Dec. 18, 2009, reallocation is weighted by 13/365 for the 13 remaining days of the calendar year until the end of the attribution analysis period. Note that the ending portfolio allocation on Dec. 31, 2009, is not taken into consideration regardless of whether there is a portfolio change on that date since it would have received a prorated weighting of zero given that such allocation would have contributed zero days toward the analysis period.

	Allocation Date	Prorated Weight	Days in Portfolio
1	Dec. 31, 2008	104 days/365 days = 28.49%	Jan. 1, 2009, to April 14, 2009
2	April 14, 2009	248 days/365 days = 67.95%	April 15, 2009, to Dec. 18, 2009
3	Dec. 18, 2009	13 days/365 days = 3.56%	Dec. 19, 2009, to Dec. 31, 2009

Following the same example for the policy benchmark average weighting calculation, let's assume that in the calendar year 2009 there was a policy change on Oct. 17, 2009, and this policy benchmark is rebalanced quarterly. This means that the policy had five updates during the analysis period, on Dec. 31, 2008, March 31, 2009, June 30, 2009, Sept. 30, 2009, and Oct. 17, 2009. Please refer to the table below:

	Allocation Date	Prorated Weight	Days in Policy Benchmark
1	Dec. 31, 2008	90 days/365 days = 24.66%	Jan. 1, 2009, to March 31, 2009
2	March 31, 2009	91 days/365 days = 24.93%	April 1, 2009, to June 30, 2009
3	June 30, 2009	92 days/365 days = 25.21%	July 1, 2009, to Sept. 30, 2009
4	Sept. 30, 2009	17 days/365 days = 4.66%	Oct. 1, 2009, to Oct. 17, 2009
5	Oct. 17, 2009	75 days/365 days = 20.55%	Oct. 18, 2009, to Dec. 31, 2009