Risk Budgeting—Where Do You Spend Your Risk?

► **Thomas M. Idzorek, CFA**
CIO & Director of Research and Product Development
Ibbotson Associates—A Morningstar Company

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What is Risk Budgeting?

▶ Risk budgeting is a quantitative endeavor that brings logic and scientific rigor to the portfolio management process that helps one to understand the risks they are taking as they attempt to maximizes returns.

▶ Risk budgeting is the process of identify, quantifying, and spending "risk" in the most efficient manner possible.
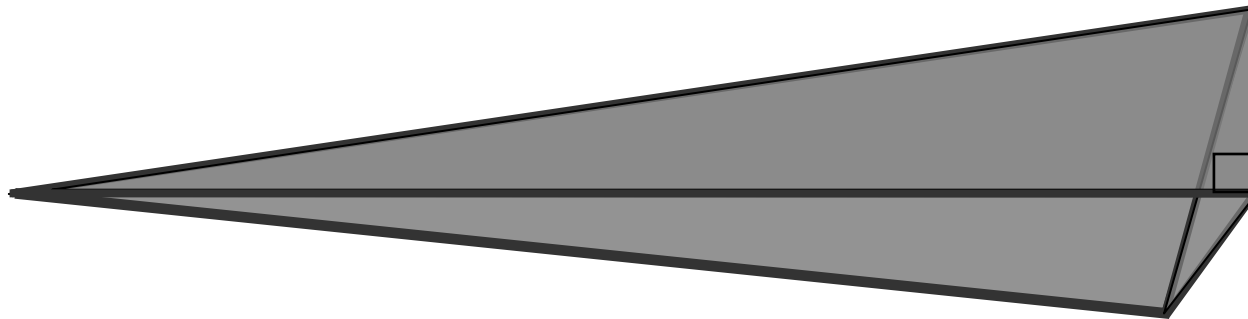
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What is Risk?

▶ There are number of different ways to define, segment, and group risk.

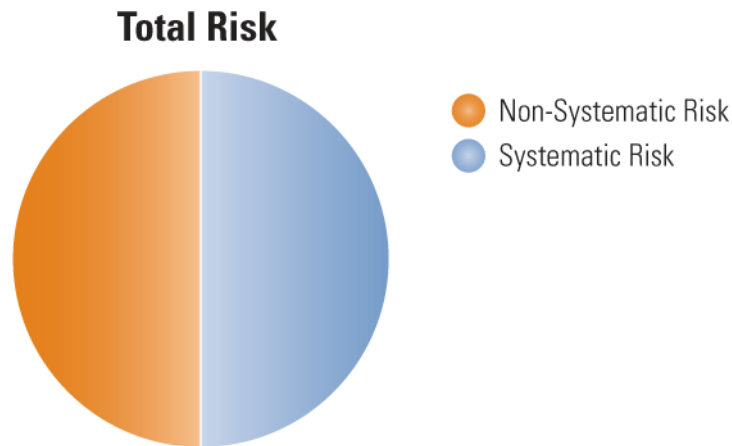▶ Total Risk, Beta Risk, Benchmark Risk, Market Risk, Active Risk, Idiosyncratic Risk etc.

Total Risk

▶ Total Risk (σ_P)–The standard deviation of your entire portfolio

Total Risk

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# Total Risk–Systematic Risk vs. Non-Systematic Risk

► Systematic Risk–Risk created by exposure to systematic (market) factors (e.g. asset class exposures)

► Non-Systematic Risk (ω_P)–Risk that is uncorrelated to systematic (market) factors (e.g. asset class exposures)

Total Risk

● Non-Systematic Risk
● Systematic Risk

For illustration only.

Total Risk–Benchmark Risk vs. Active Risk

▶ Benchmark Risk (σ_B)–The risk inherent in the benchmark; the standard deviation of the benchmark

▶ Active Risk (ψ_P)–The risk caused by holdings that differ from those of the benchmark. Active risk is a mixture of Systematic Risk and Non-Systematic Risk

Total Risk

● Benchmark Risk
● Active Risk

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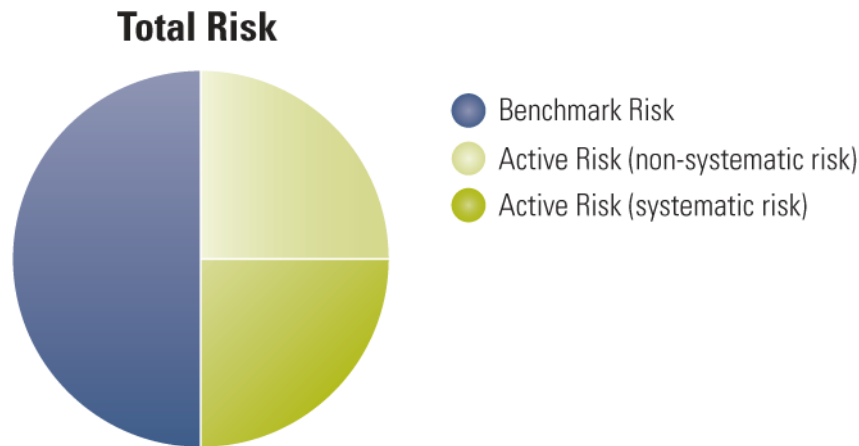
Total Risk–Benchmark Risk vs. Active Risk

▶ Benchmark Risk (σ_B)–The risk inherent in the benchmark; the standard deviation of the benchmark
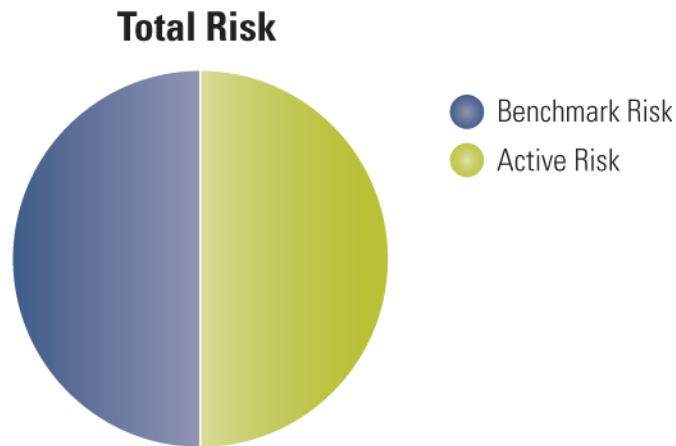
▶ Active Risk (ψ_P)–The risk caused by holdings that differ from those of the benchmark. Active risk is a mixture of Systematic Risk and Non-Systematic Risk

Total Risk

● Benchmark Risk
● Active Risk (non-systematic risk)
● Active Risk (systematic risk)
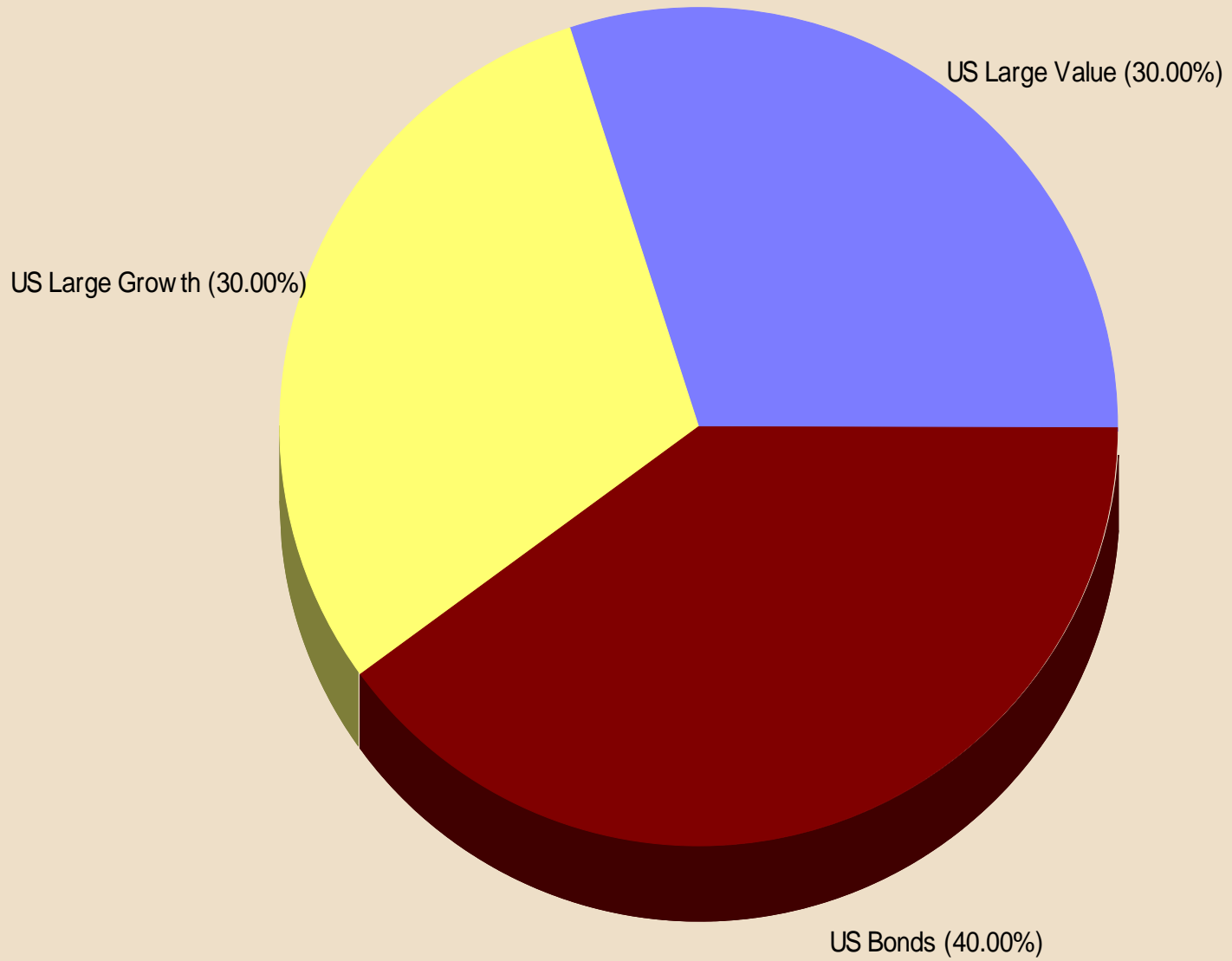
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Defining Benchmark

▶ Single Factor Benchmark (e.g. S&P 500, MSCI World, etc.)

▶ Multi-Factor Benchmarks ($\mathbf{h}_B$)

    ▶ 30% US Large Value

    ▶ 30% US Large Growth

    ▶ 40% US Bonds

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The Risk Pyramid

Benchmark Risk:

Total Risk = Benchmark Risk = Systematic Risk = 6.7%

Benchmark

US Large Value (30.00%)

US Large Growth (30.00%)

US Bonds (40.00%)

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EnCorr Optimizer - Risk Budgeting Optimizer File.aax

File   Edit   View   Frontier   Comparisons   Forecast   Simulation   Window   Help

Portfolio Statistics

Statistics... | Benchmark ▼ |

	Position 3	Benchmark	Benchmark 2	Global Mix	Global Mix 2	TAA (L v S)1	TAA (L v S)2	
US Large Value	3.7879	30.0000	20.0000	15.0000	25.0000	30.0000	25.0000	
US Large Growth	0.0000	30.0000	20.0000	15.0000	25.0000	30.0000	25.0000	
US Small Value	0.2438	0.0000	10.0000	7.5000	6.0000	0.0000	5.0000	
US Small Growth	8.9767	0.0000	10.0000	7.5000	5.0000	0.0000	5.0000	
Non-US Equities	9.7175	0.0000	10.0000	15.0000	6.0000	10.0000	10.0000	
Emerging Markets	2.4895	0.0000	0.0000	5.0000	4.0000	0.0000	0.0000	
Global Hi-Yield	0.0000	0.0000	0.0000	5.0000	3.0000	0.0000	0.0000	
US Bonds	74.7846	40.0000	30.0000	25.0000	20.0000	30.0000	30.0000	
Non-US Bonds	0.0000	0.0000	0.0000	5.0000	6.0000	0.0000	0.0000	
Expected Return	4.5981	6.0083	6.7628	7.0254	7.0294	6.7928	6.7778	
Standard Deviation	3.1305	6.7024	8.1999	8.5699	8.7437	8.2805	8.1817	
Threshold	2.6270	N/A	N/A	N/A	N/A	N/A	N/A	
Probability	73.5524	N/A	N/A	N/A	N/A	N/A	N/A	

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Calculating the Risk of the Benchmark

$$\sigma_B = \sqrt{\mathbf{h}_B^T \Sigma \mathbf{h}_B}$$

Opportunity Set	Benchmark Holdings (h_B)	Covariance Matrix (Σ)									Benchmark Holdings (h_B)
US Large Value	30.00%	0.014	0.006	0.014	0.010	0.011	0.015	0.005	-0.001	-0.001	30.00%
US Large Growth	30.00%	0.006	0.032	-0.002	0.030	0.024	0.030	0.006	-0.003	0.000	30.00%
US Small Value	0.00%	0.014	-0.002	0.026	0.009	0.009	0.009	0.006	-0.001	-0.002	0.00%
US Small Growth	0.00%	0.010	0.030	0.009	0.039	0.028	0.039	0.010	-0.005	-0.003	0.00%
Non-US Equities	0.00%	0.011	0.024	0.009	0.028	0.030	0.034	0.006	-0.004	0.001	0.00%
Emerging Markets	0.00%	0.015	0.030	0.009	0.039	0.034	0.069	0.012	-0.006	-0.002	0.00%
Global Hi-Yield	0.00%	0.005	0.006	0.006	0.010	0.006	0.012	0.008	0.000	0.001	0.00%
US Bonds	40.00%	-0.001	-0.003	-0.001	-0.005	-0.004	-0.006	0.000	0.001	0.002	40.00%
Non-US Bonds	0.00%	-0.001	0.000	-0.002	-0.003	0.001	-0.002	0.001	0.002	0.005	0.00%

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Calculating the Risk of the Benchmark

$$\sigma_B = \sqrt{\mathbf{h}_B^T \Sigma \mathbf{h}_B}$$

$$
6.7\% = \left(\begin{bmatrix} 30.00\% \\ 30.00\% \\ 0.00\% \\ 0.00\% \\ 0.00\% \\ 0.00\% \\ 0.00\% \\ 40.00\% \\ 0.00\% \end{bmatrix}^T \begin{bmatrix} 0.014 & 0.006 & 0.014 & 0.010 & 0.011 & 0.015 & 0.005 & -0.001 & -0.001 \\ 0.006 & 0.032 & -0.002 & 0.030 & 0.024 & 0.030 & 0.006 & -0.003 & 0.000 \\ 0.014 & -0.002 & 0.026 & 0.009 & 0.009 & 0.009 & 0.006 & -0.001 & -0.002 \\ 0.010 & 0.030 & 0.009 & 0.039 & 0.028 & 0.039 & 0.010 & -0.005 & -0.003 \\ 0.011 & 0.024 & 0.009 & 0.028 & 0.030 & 0.034 & 0.006 & -0.004 & 0.001 \\ 0.015 & 0.030 & 0.009 & 0.039 & 0.034 & 0.069 & 0.012 & -0.006 & -0.002 \\ 0.005 & 0.006 & 0.006 & 0.010 & 0.006 & 0.012 & 0.008 & 0.000 & 0.001 \\ -0.001 & -0.003 & -0.001 & -0.005 & -0.004 & -0.006 & 0.000 & 0.001 & 0.002 \\ -0.001 & 0.000 & -0.002 & -0.003 & 0.001 & -0.002 & 0.001 & 0.002 & 0.005 \end{bmatrix} \begin{bmatrix} 30.00\% \\ 30.00\% \\ 0.00\% \\ 0.00\% \\ 0.00\% \\ 0.00\% \\ 0.00\% \\ 40.00\% \\ 0.00\% \end{bmatrix} \right)^{1/2}
$$

The Marginal Contribution to Risk

► The marginal contribution to a type of risk is the partial derivative of the risk in question (total risk, active risk, or residual risk) with respect to the applicable type of portfolio holdings (total holdings, active holdings, or residual holdings).

► Knowing a positions marginal contribution to risk allows one to:

    ► approximate the change in portfolio risk (total risk, active risk, or residual risk) do to a change in an individual holding

    ► determine which positions are least optimal

    ► create a risk budget

The Marginal Contribution to Total Risk (MCTR)

$$MCTR = \frac{\Sigma h_B}{\sigma_B}$$

Opportunity Set	Marginal Contribution to Total Risk (MCTR)
US Large Value	8.55%
US Large Growth	15.37%
US Small Value	4.61%
US Small Growth	14.91%
Non-US Equities	13.52%
Emerging Markets	16.83%
Global Hi-Yield	4.66%
US Bonds	-1.18%
Non-US Bonds	0.36%

=

0.014	0.006	0.014	0.010	0.011	0.015	0.005	-0.001	-0.001	30.00%
0.006	0.032	-0.002	0.030	0.024	0.030	0.006	-0.003	0.000	30.00%
0.014	-0.002	0.026	0.009	0.009	0.009	0.006	-0.001	-0.002	0.00%
0.010	0.030	0.009	0.039	0.028	0.039	0.010	-0.005	-0.003	0.00%
0.011	0.024	0.009	0.028	0.030	0.034	0.006	-0.004	0.001	0.00%
0.015	0.030	0.009	0.039	0.034	0.069	0.012	-0.006	-0.002	0.00%
0.005	0.006	0.006	0.010	0.006	0.012	0.008	0.000	0.001	0.00%
-0.001	-0.003	-0.001	-0.005	-0.004	-0.006	0.000	0.001	0.002	40.00%
-0.001	0.000	-0.002	-0.003	0.001	-0.002	0.001	0.002	0.005	0.00%

6.7%

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Approximating the Change In Total Risk (Benchmark)

Opportunity Set	Marginal Contribution to Total Risk (MCTR)	Approximate Change in Risk from 1% Increase in Holding	Approximate Change in Risk from 1% Decrease in Holding
US Large Value	8.55%	0.09%	-0.09%
US Large Growth	15.37%	0.15%	-0.15%
US Small Value	4.61%	0.05%	-0.05%
US Small Growth	14.91%	0.15%	-0.15%
Non-US Equities	13.52%	0.14%	-0.14%
Emerging Markets	16.83%	0.17%	-0.17%
Global Hi-Yield	4.66%	0.05%	-0.05%
US Bonds	-1.18%	-0.01%	0.01%
Non-US Bonds	0.36%	0.00%	0.00%

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Absolute Contribution to Total Risk (Benchmark)

Opportunity Set	Marginal Contribution to Total Risk (MCTR)		Benchmark Holdings (h_B)		Absolute Contribution to Risk
US Large Value	8.55%	X	30.00%	=	2.56%
US Large Growth	15.37%	X	30.00%	=	4.61%
US Small Value	4.61%	X	0.00%	=	0.00%
US Small Growth	14.91%	X	0.00%	=	0.00%
Non-US Equities	13.52%	X	0.00%	=	0.00%
Emerging Markets	16.83%	X	0.00%	=	0.00%
Global Hi-Yield	4.66%	X	0.00%	=	0.00%
US Bonds	-1.18%	X	40.00%	=	-0.47%
Non-US Bonds	0.36%	X	0.00%	=	0.00%
					6.70%

For illustration only.

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Percentage Contribution to Total Risk

Opportunity Set	Absolute Contribution to Risk		Total Risk		Relative Contribution to Risk
US Large Value	2.56%	/	6.70%	=	38.27%
US Large Growth	4.61%	/	6.70%	=	68.80%
US Small Value	0.00%	/	6.70%	=	0.00%
US Small Growth	0.00%	/	6.70%	=	0.00%
Non-US Equities	0.00%	/	6.70%	=	0.00%
Emerging Markets	0.00%	/	6.70%	=	0.00%
Global Hi-Yield	0.00%	/	6.70%	=	0.00%
US Bonds	-0.47%	/	6.70%	=	-7.07%
Non-US Bonds	0.00%	/	6.70%	=	0.00%

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Bar Graph

Total Risk Benchmark
(Percentage)
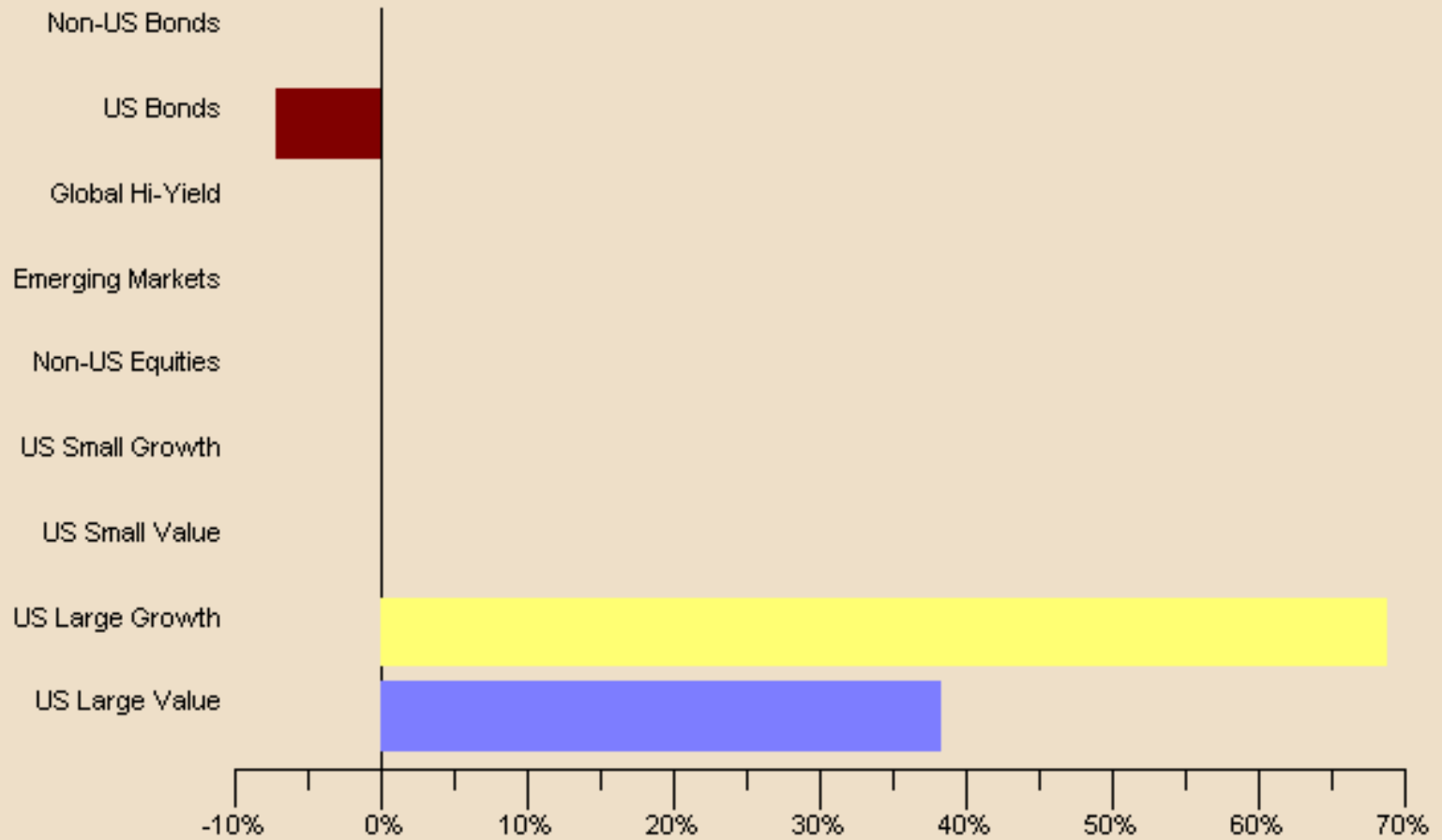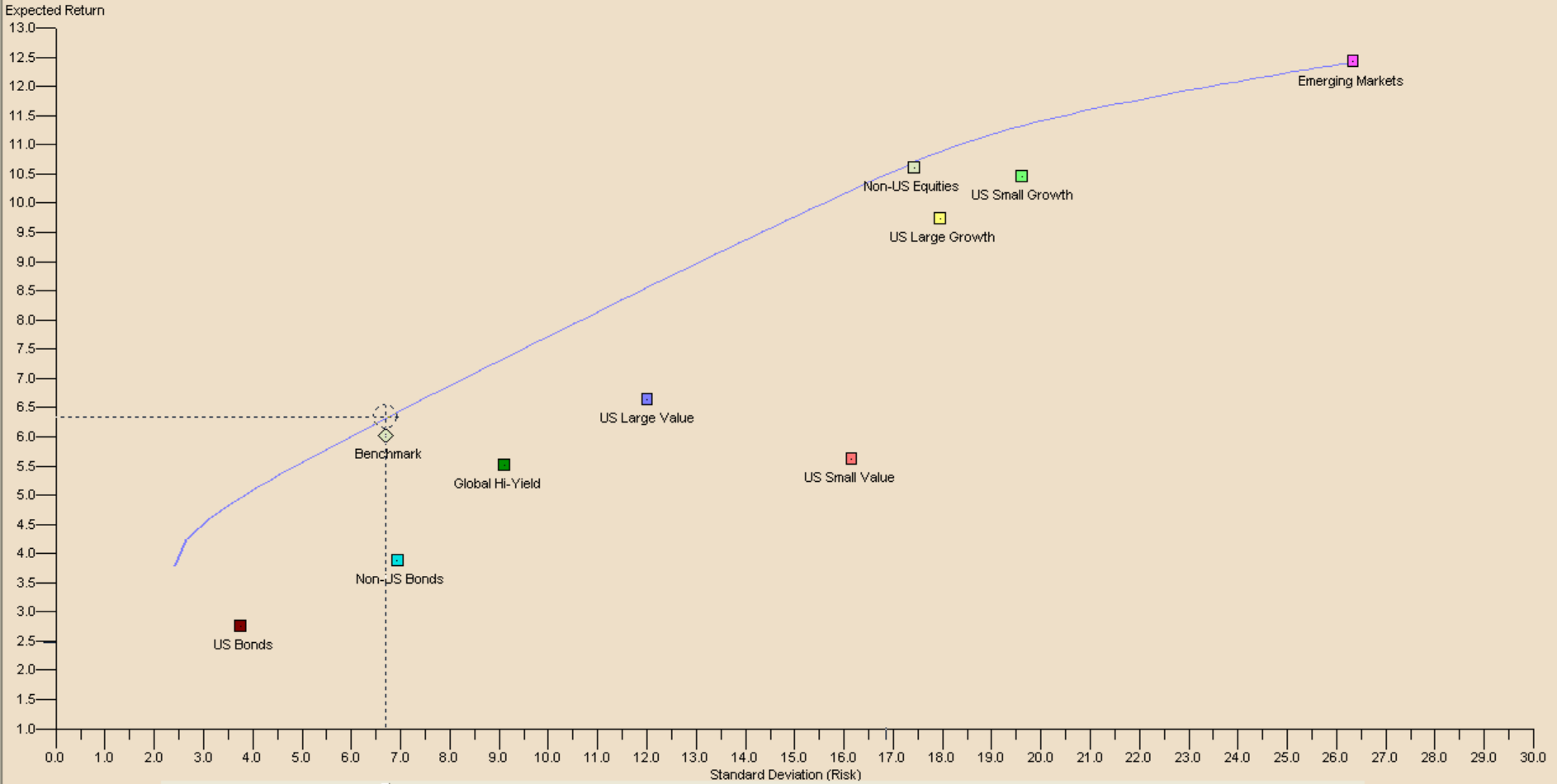
Categories (top to bottom):
- Non-US Bonds
- US Bonds
- Global Hi-Yield
- Emerging Markets
- Non-US Equities
- US Small Growth
- US Small Value
- US Large Growth
- US Large Value

X-axis: -10%, 0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%

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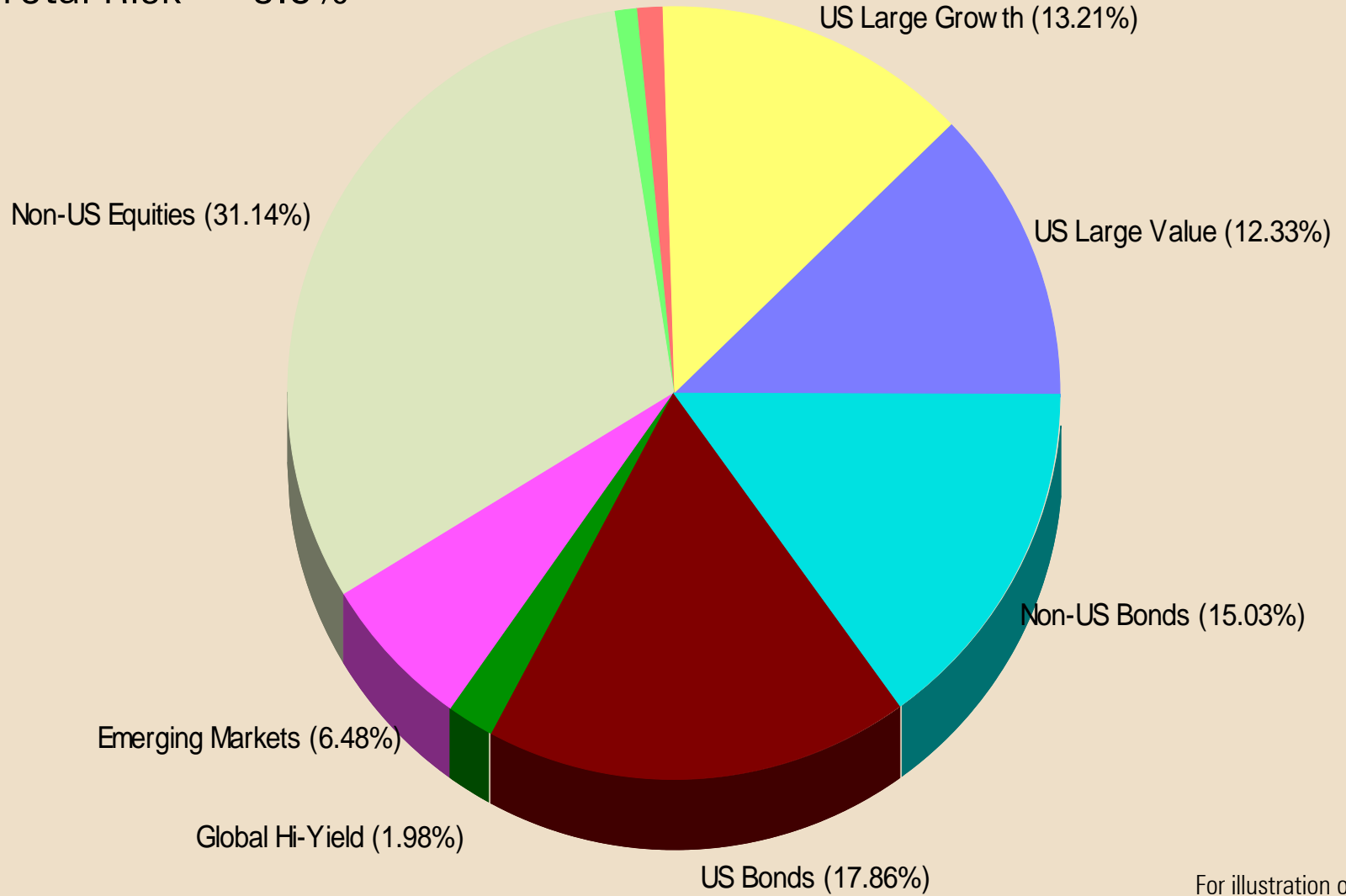
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Calculating the Risk of the Portfolio

$$\sigma_P = \sqrt{\mathbf{h}_P^T \Sigma \mathbf{h}_P}$$

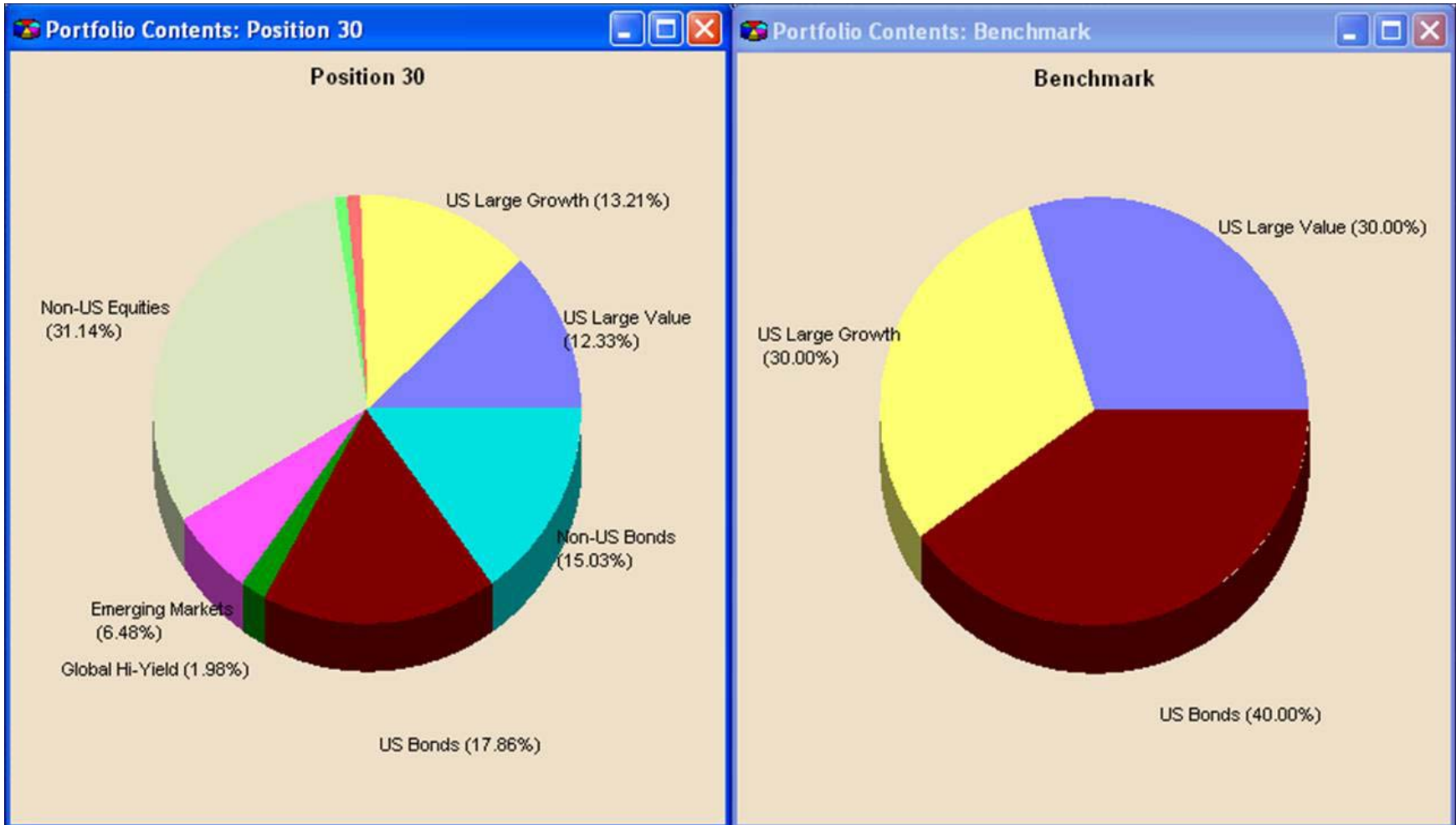| Opportunity Set	Portfolio Holdings (h_P)	Covariance Matrix (Σ)									Portfolio Holdings (h_P)
US Large Value	12.33%	0.014	0.006	0.014	0.010	0.011	0.015	0.005	-0.001	-0.001	12.33%
US Large Growth	13.21%	0.006	0.032	-0.002	0.030	0.024	0.030	0.006	-0.003	0.000	13.21%
US Small Value	1.06%	0.014	-0.002	0.026	0.009	0.009	0.009	0.006	-0.001	-0.002	1.06%
US Small Growth	0.92%	0.010	0.030	0.009	0.039	0.028	0.039	0.010	-0.005	-0.003	0.92%
Non-US Equities	31.14%	0.011	0.024	0.009	0.028	0.030	0.034	0.006	-0.004	0.001	31.14%
Emerging Markets	6.48%	0.015	0.030	0.009	0.039	0.034	0.069	0.012	-0.006	-0.002	6.48%
Global Hi-Yield	1.98%	0.005	0.006	0.006	0.010	0.006	0.012	0.008	0.000	0.001	1.98%
US Bonds	17.86%	-0.001	-0.003	-0.001	-0.005	-0.004	-0.006	0.000	0.001	0.002	17.86%
Non-US Bonds	15.03%	-0.001	0.000	-0.002	-0.003	0.001	-0.002	0.001	0.002	0.005	15.03%

Total Portfolio Risk = 9.6%

The Risk Pyramid

Benchmark Risk:

Total Risk = Benchmark Risk = Systematic Risk = 6.7%

Portfolio Risk: The Systematic Risk Triangle

Total Risk = Systematic Risk = 9.6%

Misfit Risk = 4.8%

Benchmark Risk = 6.7%
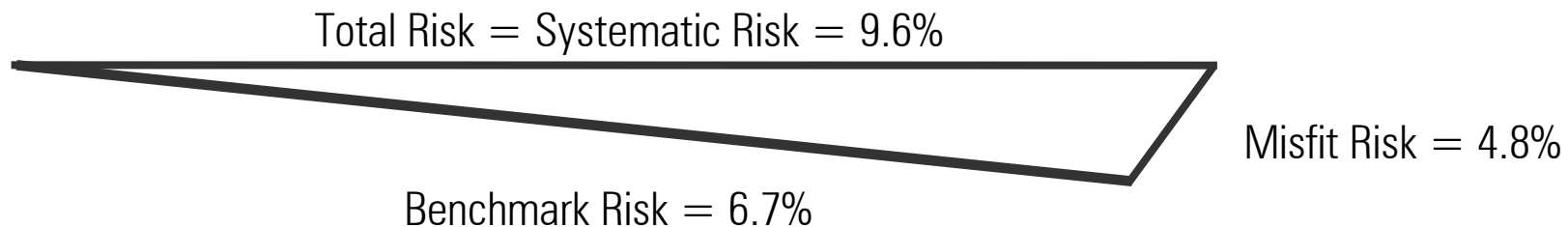
Asset Allocation "Misfit Risk"

Asset Allocation "Misfit Risk"

Opportunity Set	Portfolio Holdings (h_P)		Benchmark Holdings (h_B)		Active Holdings (h_A)
US Large Value	12.33%	—	30.00%	=	-17.67%
US Large Growth	13.21%	—	30.00%	=	-16.79%
US Small Value	1.06%	—	0.00%	=	1.06%
US Small Growth	0.92%	—	0.00%	=	0.92%
Non-US Equities	31.14%	—	0.00%	=	31.14%
Emerging Markets	6.48%	—	0.00%	=	6.48%
Global Hi-Yield	1.98%	—	0.00%	=	1.98%
US Bonds	17.86%	—	40.00%	=	-22.14%
Non-US Bonds	15.03%	—	0.00%	=	15.03%

Calculating the Misfit Risk or Active Risk

$$\sigma_A = \sqrt{\mathbf{h}_A^T \Sigma \mathbf{h}_A}$$

| Opportunity Set	Active Holdings (h$_A$)	Covariance Matrix (Σ)									Active Holdings (h$_A$)
US Large Value	-17.67%	0.014	0.006	0.014	0.010	0.011	0.015	0.005	-0.001	-0.001	-17.67%
US Large Growth	-16.79%	0.006	0.032	-0.002	0.030	0.024	0.030	0.006	-0.003	0.000	-16.79%
US Small Value	1.06%	0.014	-0.002	0.026	0.009	0.009	0.009	0.006	-0.001	-0.002	1.06%
US Small Growth	0.92%	0.010	0.030	0.009	0.039	0.028	0.039	0.010	-0.005	-0.003	0.92%
Non-US Equities	31.14%	0.011	0.024	0.009	0.028	0.030	0.034	0.006	-0.004	0.001	31.14%
Emerging Markets	6.48%	0.015	0.030	0.009	0.039	0.034	0.069	0.012	-0.006	-0.002	6.48%
Global Hi-Yield	1.98%	0.005	0.006	0.006	0.010	0.006	0.012	0.008	0.000	0.001	1.98%
US Bonds	-22.14%	-0.001	-0.003	-0.001	-0.005	-0.004	-0.006	0.000	0.001	0.002	-22.14%
Non-US Bonds	15.03%	-0.001	0.000	-0.002	-0.003	0.001	-0.002	0.001	0.002	0.005	15.03%

Active Risk = 4.8%

The Risk Pyramid

Benchmark Risk:

Total Risk = Benchmark Risk = Systematic Risk = 6.7%

Portfolio Risk: The Total Risk Pyramid (Tetrahedron)

Active Risk = X.X%

Total Risk = X.X%

Residual Risk = X.X%

Systematic Risk = 9.6%

Misfit Risk = 4.8%
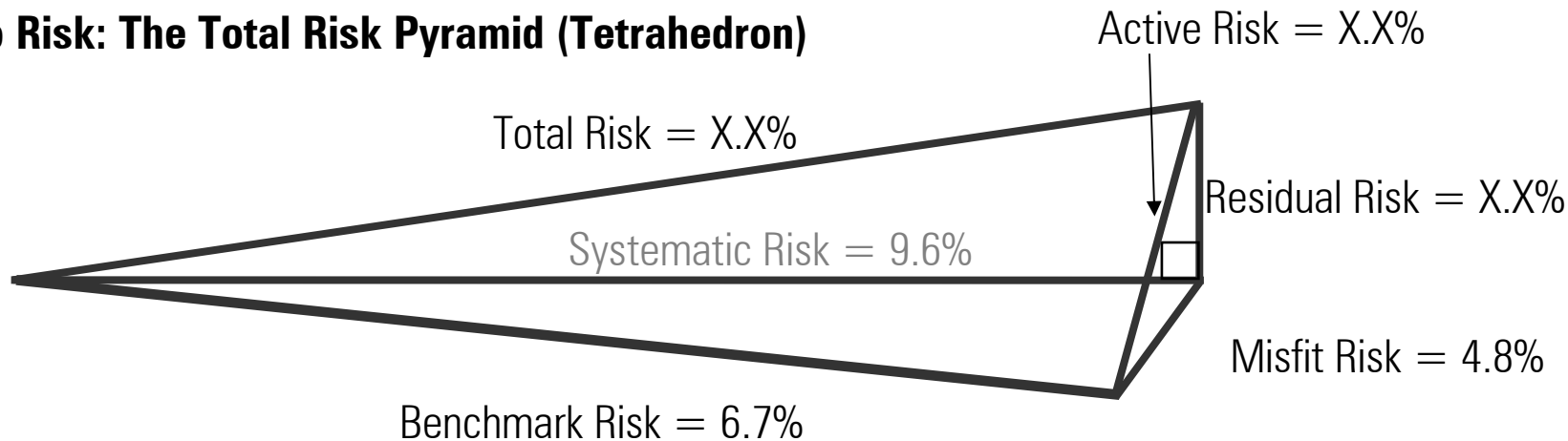
Benchmark Risk = 6.7%

The Risk Pyramid

Benchmark Risk:

Total Systematic Risk = 9.6%

Total Risk = Benchmark Risk = Systematic Risk = 6.7%

Benchmark Risk = 6.7%

Misfit Risk = 4.9%

Total Risk = X.X%

Residual Risk = X.X%

Portfolio Risk: The Total Risk Pyramid (Tetrahedron)

Total Systematic Risk = 9.6%
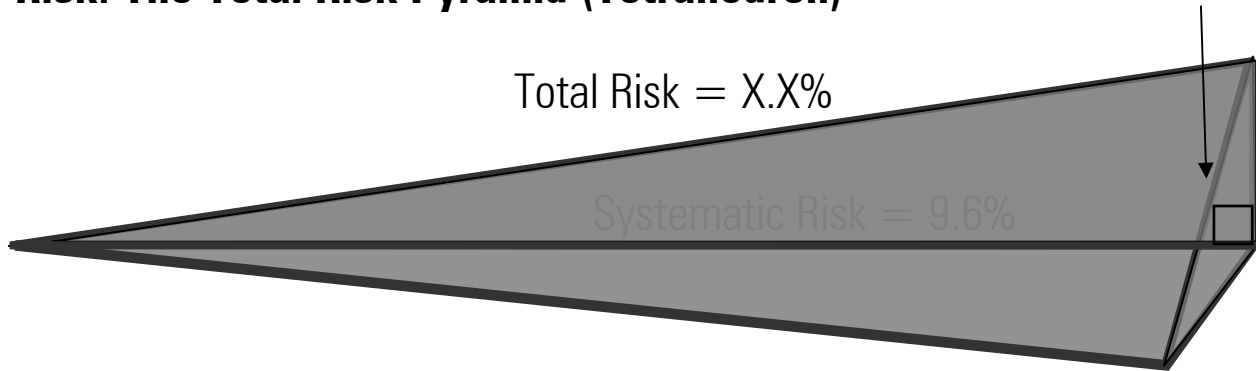
Active Risk = X.X%

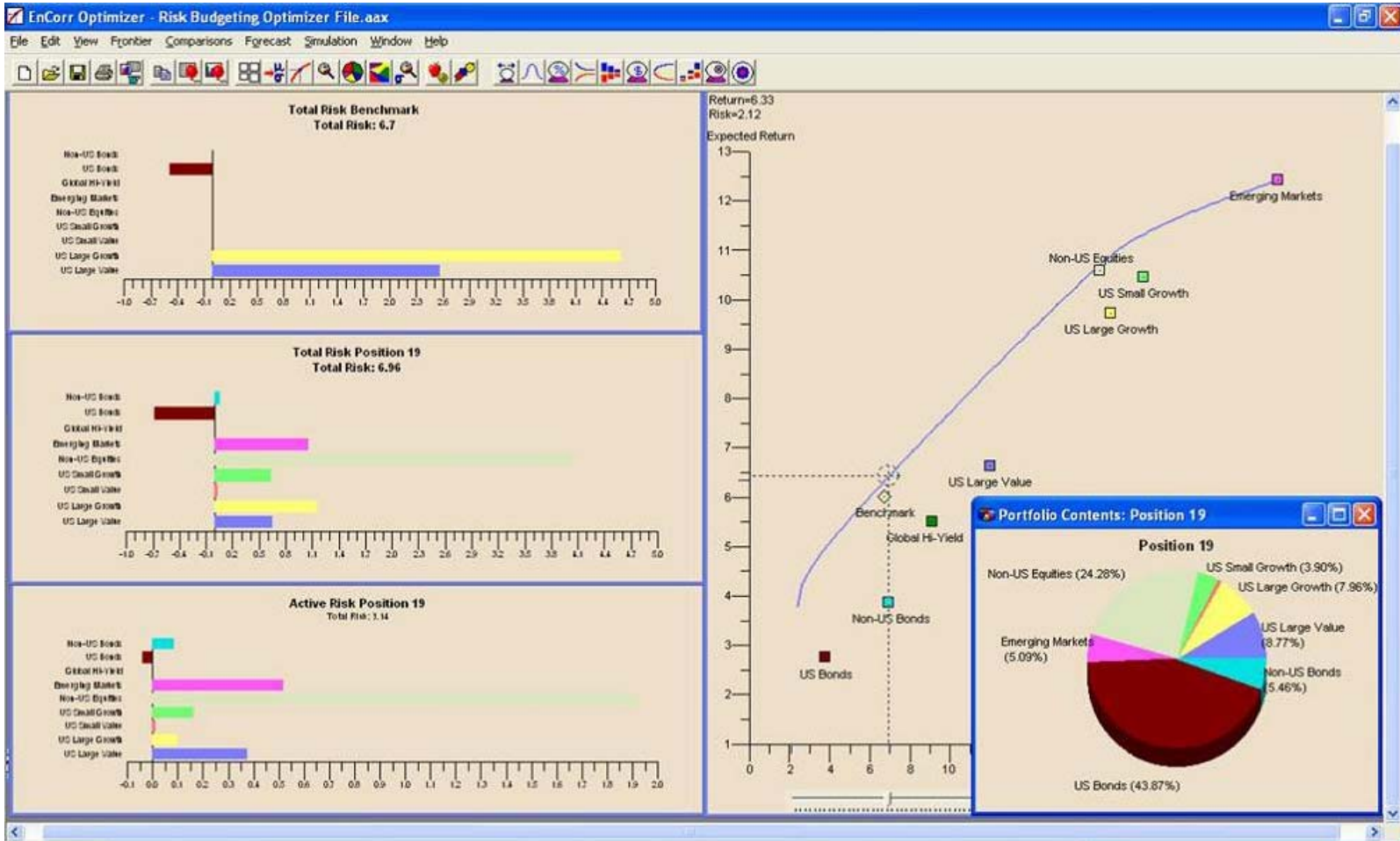Total Risk = X.X%

Residual Risk = X.X%

Systematic Risk = 9.6%

Misfit Risk = 4.9%

Benchmark Risk =6.7%

Risk Dashboard: Morningstar® EnCorr® 9.4

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Usage Examples:

► Better Asset Mix that Tracks 60/40 benchmark with 300 bps

  ► Select From Frontier

► Sizing Tactical Asset Allocation Bets

  ► View: Over weight large caps & under weight small caps with a maximum of 200 bps of active risk

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Reference(s)

▶ Grinold, Richard C., and Ronald N. Kahn. (1999). *Active Portfolio Management*. 2nd ed. New York: McGraw-Hill.

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▶ Winkelmann, Kurt. (2004). "Improving Portfolio Efficiency." *The Journal of Portfolio Management*, Winter, 23-38.

Disclosures

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